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1.01 Systems of units

All the quantities in terms of which laws of Physics are described and whose measurement is necessary are called physical quantities

For example length, mass, time, electric current, temperature, amount of substance, luminous intensity etc.

The chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity is called the unit of the quality

For example when we talk about length then unit of length is meter or kg represents mass of the object.

When we say 5 kg which is equals to 5 times of 1kg. Thus it is basically a comparison process

1.02 Fundamental and Derived unit

Fundamental unit, is a unit adopted for measurement of a fundamental quantity. A fundamental quantity is one of a conventionally chosen subset of physical quantities, where no subset quantity can be expressed in terms of the others. In the International System of Units there are fundamental units

Fundamental and Supplementary units of SI

Sr .No	Physical fundamental quantity	Fundamental units	Symbol
1	Mass	Kilogram	kg
2	Length	Metre	m
3	Time	Second	s
4	Electric current	Ampere	A

5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd
7	Quantity of matter	mole	mol
Sr.No	Supplementary physical quantity	Supplementary unit	Symbol
1	Plane angle	Radian	rad
2	Solid angle	Steradian	sr

Choice of a standard unit

The unit chosen for measurement of nay physical quantity should statisfy following requirements

i) It should be of suitable size

ii) It should be accurately defined

iii) It should be easily accessible iv) Replicas of unit should be available easily

iv) It should not change with time

v) It should not change with change in physical condition like temperature, pressure etc

1.03 Systems of units

a) The f.p.s system is the British engineering system of units, which uses foot as the unit of length, pound as the unit of mass and second as the unit of time.

b) The c.g.s system is the Gaussian system which uses the centimeter, gram and second as the three basic units.

c) The M.K.S system is based on metre, kilogram and second as the three basic units

d) International system of unit (SI) This system is introduced in 1960, by the General Conference of Weight and Measures. This system of units is essentially a modification over the m.k.s. system

Following are advantages of SI over all other system

1. SI system assigns only one unit to a particular quantity. For example joule is the unit for all types of energy, while in MKS system joule is unit for energy and calories is unit for heat energy.

2. SI system follows decimal system i.e. the multiples and submultiples of units are expressed as power of 10

3. SI system is based on certain fundamental units, from which all derived units are obtained by multiplying or division without introducing numerical factors

1.04 Some important Practical Units

1. Astronomical unit (AU)

It is the average distance of the centre of the sun from the centre of the earth

1AU $=1.496\times10^{11}$ m $\cong 1.5\times10^{11}$ m

2. Light Year (ly)

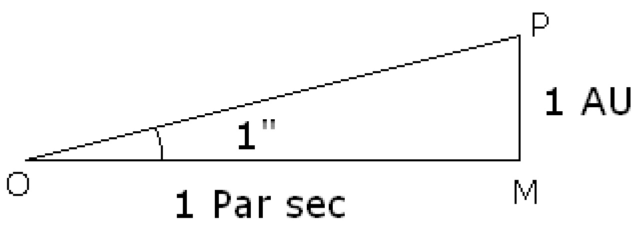
One light year is the distance travelled by light in vacuum in one year.

1 light year = $3\times10^{8}\times$ ($365\times24\times60\times60$) meter

1ly = 9.46×10^{15} m

3. Par sec.

One Par sec is the distance at which an arc 1AU long subtends an angle of 1″ (one second)



Conversion of 1 Par sec in metre

Since angle is very small tanθ =θ = PM/OM

1″ = 1 AU/ 1 Par sec

1 Par sec = 1AU/1″ -- -- eq(1)

We know that $1'' = \frac{\pi}{180\times60\times60}\,rad$ and 1AU =1.496×10^{11} m

Substituting values of 1″ and 1AU in eq(1) and on simplification we get

1 Par sec = 3.1×10^{16} m

4. Relation between AU, ly and par sec

1 ly = 6.3×10^{4} AU

1 parsec = 3.26 ly

1 Å (angstrom) = 10^{-12} m

1 amu = 1.66×10^{-27} kg

Metric prefixes

Sr.No	Power of 10	Prefix	Symbol
1	10^{-1}	deci	d
2	10^{-2}	centi	c
3	10^{-3}	milli	m

Sr.No	Power of 10	Prefix	Symbol
9	10^{1}	deca	da
10	10^{2}	hecto	h
11	10^{3}	kilo	k

4	10^{-6}	micro	μ
5	10^{-9}	nano	n
6	10^{-12}	pico	p
7	10^{-15}	femto	F
8	10^{-18}	atto	a

12	10^{6}	mega	M
13	10^{9}	giga	G
14	10^{12}	tera	T
15	10^{15}	peta	P
16	10^{18}	exa	e

Exercise 1.01

1. How many astronomical units are there in 1 metre?

2. Calculate the number of light years in one meter

3. How many amu make 1 kg

Answers : 1) 6.68×10^{-12} AU, 2) 1.057×10^{-16} ly 3) 0.6×10^{27} amu

1.05 SI Derived Units

Derived units are units which may be expressed in terms of base units by means of mathematical symbols of multiplication and division.

Certain derived units have been given special names and symbols, and these special names and symbols may themselves be used in combination with the SI and other derived units to express the units of other quantities.

Following are the few examples

SrNo	Physical quantity	Relation with other quantities	SI unit

1	Area	Length × breadth	m^2
2	Density	$\frac{mass}{volume}$	kg m^{-3}
3	Speed or velocity	$\frac{displacement}{time}$	ms^{-1}
4	Linear momentum	Mass × velocity	$Kgms^{-1}$
5	Acceleration	$\frac{change\ in\ velocity}{time}$	ms^{-2}
6	Force	Mass × acceleration	$Kgms^{-2}$ or N
7	Moment of force (torque)	Force × displacement	N-m
8	Work	Force × displacement	J (joule)
9	Power	$\frac{work}{time}$	Js^{-1} or W(watt)

1.6 Significant figure

In scientific work all numbers are assumed to be derived from measurements and therefore the last digit in each number is uncertain. All certain digits plus the first uncertain digit are significant.

For example if we measure a distance using metre scale. Least count of metre scale is 0.1 cm. Now if we measure a length of rod and it is between 47.6 cm and 47.7cm then we may estimate as 47.68 cm. Now this expression has 3 significant figure 4,7,6 are precisely known but last digit 8 is only approximately known.

Common rules for counting significant figure

Rule 1: All nonzero digits are significant

Example: x= 2365 have four significant digits

Rule 2: All the zeros between two nonzero digits are significant no matter where the decimal point is it at all.

Example: X = 1007 has four significant digits, Where as x = 2.0807 have five significant digit

Rule3: If the number is less than 1 then zeros on the right of decimal point but to the left of the first nonzero digit are not significant.

Example: X = 0.0057 has only two significant digits, but x = 1.0057 have five significant digits according to Rule2

Rule4: All zeros on the right of the last non zero digit in the decimal part are significant

Example: X = 0.00020 have two significant digits

Rule5: All zeros on the right of non-zero digit are not significant

Example: X = 8000 have only one significant digit while x = 32000 have only two significant digits

Rule6: All zeros on the right of the last nonzero digit become significant, when they come from a measurement. Also note that change in the units of measurement of a quantity does not change the number of significant digits.

Example: If measured quantity is 2030 m then number has 4 significant digits. Same can be converted in cm as 2.030×10^5 cm here also number of significant digits is to be four.

Illustration

7) State the number of significant figures in the following: (i) 600900 (ii) 5212.0 (iii) 6.320 (iv) 0.0631 (v) 2.64×10^{24}

Answers

i) 4 ii) 5 iii) 4 iv) 3 v) 3

1.06.01 Round off

Rule1: If the digit to be dropped is less than 5, then the preceding digit is left unchanged. Example 5.72 round-off to 5.7

Rule 2: If the digit to be dropped is more than 5, then the preceding digit is increased by one. Example 5.76 round-off to 5.8

Rule3: If the digit to be dropped is 5 followed by nonzero number, then preceding digit is increased by one Example: 13.654 round-off to 13.7

Rule4: If the digit to be dropped is 5, then preceding digit is left unchanged, if even Example $4.\underline{2}50$ or $4.\underline{2}52$ becomes 4.2

Rule5: If the digit to be dropped is 5, then the preceding digit is increased by one, if it is odd. Example $4.\underline{3}50$ or $4.\underline{3}52$ becomes 4.4

Illustration

8) Round off the following numbers to three significant digits (a) 15462 (b) 14.745 (c)14.750 (d) 14.650×10^{12}

Solution

(a) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore, be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 become 15500 on rounding to three significant digits.

(b) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 become 14.7 on rounding to three significant digits.

(c) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

1.06.02 Arithmetical operations with significant figure

Addition and subtraction

In addition or subtraction, the number of decimal places in the result should the <u>smallest number of decimal places</u> of terms in the operation

Example1 : The sum of three measurements 2.2 m, 2.22m, 2.222m is 6.642 round off is 6.6m

Example 2: If x= 2.35 and y = 2.1 the x-y = 0.25 Round off to 0.2 (as per round-off rule 2)

Multiplication and division

In multiplication and division, the number of significant figure in the product or in the quotient is the same as the smallest number of digits in any of the factor

Example 1: If x= 2.35 and y = 2.1 then xy =4.935 round off will be 4.9 as least significant digits is 2 in 2.1

Example 2: If x = 2500 and y = 123 then x/y =20.3252 round-off 20 as 2500 have only two significant numbers.

Illustration

9) Multiply 2.2 and 0.225. Give the answer correct to significant figures.

Solution: $2.2 \times 0.225 = 0.495$ since the least number of significant figures in the given data is 2, the result should also have only two significant figures.

∴ $2.2 \times 0.225 = 0.50$

10) Find the value of π^2 correct to significant figures, if $\pi = 3.14$

Solution

$\pi^2 = 3.14 \times 3.14 = 9.8596$

Since the least number of significant figure in the given data is 3, the result should also have only three significant figures $\pi^2 = 9.86$ (rounded off)

Exercise 1.02

Q1) 5.74 g of a substance occupies a volume of 1.2 cm^3. Calculate its density applying the principle of significant figures.

Q2) The length, breadth and thickness of a rectangular plate are 4.234 m, 1.005 m and 2.01 cm respectively. Find the total area and volume of the plate to correct significant figures.

Answers

1) 4.8 g cm^{-3} 2) 4.255 m^2 ,0.0855 m^3

1.07 Errors of measurement

Measurement cannot be perfect as the errors involved in the process cannot be removed completely. Difference between measured value and true value is called error of measurement. The error in measurement are classified as Systemic errors, Random errors and Gross errors

Systemic error

Instrum ental error: It may be due to manufacturing defect of instrument, there may be zero error.

Personal error: May be due to inexperience of the observer. For example improper setting of instrument, not following proper method of taking observation.

Error due to imperfection: Arises on account of ignoring fact. For example if temperature should be at say 25^{O} C while taking observation, and if temperature is more or less than 25^{O} C then error will happen

Error due to external causes: If there is sudden radiation or temperature change which is not under your control will affect observation

Random error

The random error are those error, during repeated observation by same person, cause may be different at every time. Such error can be minimized by taking average of many readings.

Gross errors

These errors arise on account of shear carelessness of the observer. For example Reading an instrument without setting properly. Recording the observation wrongly. Using wrong values of the observations in calculation.

1.08 Absolute error, Relative error and Percentage error

1.08.01 Absolute error

Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity

Let a physical quantity be measured n times. Let the measured values be a_1, a_2, a_3, a_4, …., a_n. the arithmetic mean is

$$a_m = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

If true value is not known then arithmetic mean is taken as true value.

By definition, absolute errors in the measured vales are

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

$$\Delta a_3 = a_m - a_3$$

$$\Delta a_n = a_m - a_n$$

The absolute error may be positive or negative.

1.08.02 Mean absolute error

It is the arithmetic mean of the modulus absolute error. It is represented as $\Delta\bar{a}$

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots + |\Delta a_n|}{n}$$

Hence final result of measurement is $a = a_m \pm \Delta\bar{a}$

Illustration

11) A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then what will be the reported mean time?

Solution

Mean of reading = (90+91+95+92)/4 = 92 s

Absolute error =

$$\frac{|92-90| + |92-91| + |92-95| + |92-92|}{4} = 1.5s$$

Minimum division is 1 and absolute error is 1.5 hence reported error = 2s

Reported mean time = 92±2s

1.08.03 Relative error

The relative error is the ratio of mean absolute error to the mean value of the quantity measured

$$relative\ error = \frac{\overline{\Delta a}}{a_m}$$

$$percentage\ error = \frac{\overline{\Delta a}}{a_m} \times 100\%$$

Illustration

12) The length of a rod is measured as 25.0 cm using a scale having an accuracy of 0.1 cm. determine the percentage error in length.

Solution

Accuracy is the maximum possible error = 0.1 cm

∴ % error = (0.1/25)×100 = 0.4 %

1.9 Combination of errors

If any experiment involves many observations and involves many mathematical operation then errors in measurement gets combined. For example density is ratio of mass and volume. Here error will be in mass and length.

1. Errors in summation

Suppose z = a + b

Let Δa and Δb be the absolute error in measurement of a and b

Then $z \pm \Delta z = (a \pm \Delta a) + (b \pm \Delta b)$

$z \pm \Delta z = (a+b) \pm (\Delta a + \Delta b)$

Thus $\Delta z = \Delta a+\Delta b$

Note error gets added

2. Error in difference

Suppose $z = a - b$

Let Δa and Δb be the absolute error in measurement of a and b

Then $z\pm\Delta z = (a\pm\Delta a) - (b\pm\Delta b)$

$z\pm\Delta z = a\pm\Delta a - b\mp\Delta b$

$z\pm\Delta z = a-b \pm\Delta a \mp\Delta b$

$\pm\Delta z = \pm\Delta a \mp\Delta b$

Thus there are four possible values of error

$(+\Delta a +\Delta b), (+\Delta a -\Delta b), (-\Delta a +\Delta b), (-\Delta a -\Delta b)$

Therefore maximum error $\Delta z = \Delta a+\Delta b$

Note error gets added

3. Error in product

Let $z = a\times b$

$z \pm z = (a\pm\Delta a) \times (b\pm\Delta b)$

$z \pm z = ab \pm a\Delta b\pm\Delta ab +\Delta a\Delta b$

as Δa and Δb are very small , $\Delta a\Delta b$ can be neglected

$z \pm z = ab \pm a\Delta b\pm\Delta ab$

$\pm z = \pm(a\Delta b+\Delta ab)$

Now dividing above equation by z

$$\frac{\Delta z}{z} = \frac{a\Delta b}{z} + \frac{b\Delta a}{z}$$

But z = ab

$$\frac{\Delta z}{z} = \frac{a\Delta b}{ab} + \frac{b\Delta a}{ab}$$

$$\frac{\Delta z}{z} = \frac{\Delta b}{b} + \frac{\Delta a}{a}$$

Note relative error gets added

4. Error in division

Let $z = \frac{a}{b} = ab^{-1}$

By taking first order derivative with respect to z we get

$$1 = -ab^{-2}\frac{db}{dz} + b^{-1}\frac{da}{dz}$$

$$dz = -ab^{-2}(db) + b^{-1}(da)$$

Dividing by z

$$\frac{dz}{z} = -\frac{ab^{-2}(db)}{z} + \frac{b^{-1}(da)}{z}$$

But z = ab^{-1} and error are either positive or negative

$$\pm\frac{dz}{z} = \mp\frac{ab^{-2}(db)}{ab^{-1}} \pm \frac{b^{-1}(da)}{ab^{-1}}$$

$$\pm\frac{dz}{z} = \mp\frac{(db)}{b} \pm \frac{(da)}{a}$$

Or

Thus four possible values of errors

$$= -\frac{(db)}{b} + \frac{(da)}{a}, -\frac{(db)}{b} - \frac{(da)}{a}, +\frac{(db)}{b} - \frac{(da)}{a}, +\frac{(db)}{b} + \frac{(da)}{a}$$

Thus maximum error in terms of fractional error cab be wrote as

$$\frac{\Delta z}{z} = \left(\frac{\Delta b}{b} + \frac{\Delta a}{a}\right)$$

Note relative error gets added

5. Errors in powers

$$z = \frac{a^n}{b^m}$$

Taking long on both sides

log z = log a^n – log b^m

log z = nloga –mlogb

by taking derivative we get

$$\frac{dz}{z} = n\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error

$$\pm\frac{\Delta z}{z} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore maximum value of error

$$\frac{\Delta z}{z} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b}$$

General formula

$$z = \frac{a^n b^m}{c^p}$$

$$\frac{\Delta z}{z} = n\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta c}{c}$$

Illustration

13) In an experiment to determine acceleration due to gravity by simple pendulum, a student commit 1%positive error in the measurement of length and 3% negative error in the measurement of time period. What will be percentage error in the value of g

Solution

We know that period T

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = 4\pi\frac{l}{T^2}$$

$$\frac{\Delta g}{g}\% = \frac{\Delta l}{l}\% + 2\frac{\Delta T}{T}\%$$

$$\frac{\Delta g}{g}\% = 1\% + 2\times 3\% = 7\%$$

Thus error in gravitational acceleration = 7%

6. Error in reciprocal

If a given formula is in reciprocal form, we can determine error

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{1}{c} = \frac{a+b}{ab}$$

$$c = \frac{ab}{a+b}$$

$$\frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta(a+b)}{a+b}$$

Illustration

14) In an experiment, value of resistance of two resistance are $r_1 = (10\pm0.2)\Omega$ ohm and $r_2 = (30\pm0.4)\ \Omega$. Find the value of total resistance if connected in parallel with limit of error

Solution

Calculation of total resistance R using formula for parallel connection

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$R = \frac{r_1 r_2}{r_1 + r_2} = \frac{10\times30}{10+40} = 6\Omega$$

Using formula for relative error

$$\frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta a + \Delta b}{a+b}$$

$$\frac{\Delta R}{6} = \frac{0.2}{10} + \frac{0.4}{30} + \frac{0.2+0.4}{10+30} = \frac{2.6}{60}$$

$$\Delta R = 0.26\Omega$$

$$\therefore R = (6\pm0.6)\Omega$$

15) Calculate focal length of a spherical mirror from the following observation object distance u = 10±0.03, image distance 40±0.02

Solution:

Calculation of focal length

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$f = \frac{uv}{u+v} = \frac{10\times 40}{10+40} = 8cm$$

Using formula for relative error

$$\frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta a + \Delta b}{a+b}$$

$$\frac{\Delta f}{8} = \frac{0.03}{10} + \frac{0.02}{40} + \frac{0.03+0.02}{10+40}$$

$$\frac{\Delta f}{8} = 0.0045$$

∴Δf =0.036 cm

Thus focal length of spherical mirror = (8.0±0.036)cm

16) The time period of simple pendulum is given by $t = 2\pi\sqrt{\frac{l}{g}}$. What is the accuracy in the determination of g if 10 cm length is known to 1mm accuracy and 0.5s time period is measured from time of 100 oscillations with watch of 1 second resolution?

Solution

Total time for 100 oscillation = 100×0.5s = 50 s. Watch resolution is 1s thus Δt = 1s . 1mm = 0.1 cm

$$t = 2\pi\sqrt{\frac{l}{g}}$$

$$g = 4\pi^2\frac{l}{t^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta t}{t}$$

$$\frac{\Delta g}{g}\% = \frac{0.1}{10}\times 100 + \frac{2}{50}\times 100 = \pm 5\%$$

1.10 Accuracy and precision

A measurement's accuracy

Accuracy refers to the closeness of a measured value to a standard or known value. Problems with accuracy are due to errors. For example, if in lab you obtain a weight measurement of 3.2 kg for a given substance, but the actual or known weight is 10 kg, then your measurement is not accurate. In this case, your measurement is not close to the known value. We can necessary precaution and reduce different types of error discussed in section 1.13 on errors

Precision

Precision refers to the closeness of two or more measurements to each other. Using the example above, if you weigh a given substance five times, and get 3.2 kg each time, then your measurement is very precise. Precision is independent of accuracy. You can be very precise but inaccurate, as described above. You can also be accurate but imprecise.

Precision describes the limitation of the measuring instrument. Measurement's precision is determined by least count of the measuring instrument. Smaller the least count, greater is the precision.

1.11 Dimensional analysis

1.16.01 Dimensions of a physical quantity

The dimensions of a physical quantity are the power to which the fundamental units of mass, length and time have to be raised to represent a derived unit of quantity.

Fundamental unit of mass is represented by [M], length by [L] and time by [T].

Suppose we obtained derived unit area

As Area = length × width

Area = [L][L] = [L^2]

Thus to represent area we have to raise [L] to the power of 2. Thus area is said to have two dimensions. Similarly volume is said to have three dimensions.

Since for area and volume mass and time are not required we write

A = [$M^0L^2T^0$] and V = [$M^0L^3T^0$] are also called as dimensional formula or dimensional equation for velocity

For velocity we write

$$V = \frac{dispalcement}{ime} = \frac{[L]}{[T]}$$

Velocity = [$M^0L^1T^{-1}$] is also called as dimensional formula or dimensional equation for velocity

Hence dimension of velocity are: zero in mass, +1 in length and -1 in time.

1.11.02 Types of physical quantities

1. Dimensional constant

These are the quantities whose values are constant and they posse's dimensions. For examples: velocity of light, Planck's constant

2. Dimensional Variable

These are the quantities whose values are variable and they have dimensions For example velocity, acceleration, volume etc.

3. Dimensionless constants

These are the quantity, whose values are constant, but do not posses dimensions. Example numbers 1, 2, 3 … or mathematical constant e and π

4. Dimensionless variables

These are the quantities, whose values changes and they do not have dimensions. For example angle, solid angle, specific gravity, refractive index

1.16.03 some important dimension formula

SrNo	Physical quantity	Dimension	SI unit
1	Force (F)	$[M^1 L^1 T^{-2}]$	newton
2	Work (W)	$[M^1 L^2 T^{-2}]$	joule
3	Power(P)	$[M^1 L^2 T^{-3}]$	watt
4	Gravitational constant(G)	$[M^{-1} L^3 T^{-2}]$	$N\text{-}m^2/kg^2$
5	Angular velocity (ω)	$[T^{-1}]$	radian/s
6	Angular momentum(L)	$[M^1 L^2 T^{-1}]$	$kg\text{-}m^2/s$
7	Moment of inertia (I)	$[M^1 L^2]$	$kg\text{-}m^2$
8	Torque (τ)	$[M^1 L^2 T^{-2}]$	N-m
9	Young's modulus (Y)	$[M^1 L^{-1} T^{-2}]$	N/m^2
10	Surface Tension (S)	$[M^1 T^{-2}]$	N/m
11	Coefficient of viscosity (η)	$[M^1 L^{-1} T^{-1}]$	$N\text{-}s/m^2$
12	Pressure (p)	$[M^1 L^{-1} T^{-2}]$	N/m^2(Pascal)
13	Specific heat capacity (c)	$[L^2 T^{-2} K^{-1}]$	J/kg-K
14	Stefan's constant (σ)	$[M^1 T^{-3} K^{-4}]$	$watt/m^2\text{-}k^4$

15	Current density (j)	$[I^1 L^{-2}]$	ampere/m^2
16	Electrical conductivity (σ)	$[I^2T^3M^{-1}L^{-3}]$	$\Omega^{-1}m^{-1}$
17	Electric dipole moment (p)	$[L^1I^1T^1]$	C-m
18	Electric field (E)	$[M^1L^1I^{-1}T^{-3}]$	V/m
19	Electrical potential (V)	$[M^1L^2I^{-1}T^{-3}]$	volt
20	Electric flux (ϕ)	$[M^1L^{-3}I^{-1}T^3]$	volt/m
21	Capacitance (C)	$[M^{-1}L^{-2}I^2T^4]$	farad (F)
22	Permittivity (ε)	$[M^{-1}L^{-3}I^2T^4]$	C^2/N-m^2
23	Permeability (μ)	$[M^1L^1I^{-2}T^{-3}]$	Newton/A^2
24	Magnetic dipole moment (M)	$[L^2I^1]$	N-m/T
25	Magnetic flux (ϕ)	$[M^1L^2I^{-1}T^{-2}]$	Weber(Wb)
26	Magnetic field (B)	$[M^1I^{-1}T^{-2}]$	tesla
27	Inductance (L)	$[M^1L^2I^{-2}T^{-2}]$	henry
28	Resistance (R)	$[M^1L^2I^{-2}T^{-3}]$	ohm (Ω)
29	Intensity of wave (I)	$[M^1T^{-3}]$	watt/m^2
30	Thermal conductivity (k)	$[M^1L^1T^{-3}K^{-1}]$	watt/m-K

1.11.04 Uses of dimensional equation

Dimensional analysis is used for 1. Conversion of one system of units to another 2. Checking the accuracy of various formulae. 3. Derivation of formulae

1. Conversion of one system of units to another

Magnitude of a physical quantity remains same, whatever be the system of its measurement for example length of 1 m length rod will remains same if expressed as 100 cm. If Q is the magnitude of physical quantity, u_1 and u_2 are two units of measurement, n_1 and n_2 are their respective numerical value then

$$Q = n_1u_1 = n_2u_2$$

Let M_1, L_1 T_1 and M_2, L_2, T_2 be the fundamental units for respective system, let a, b, c are the respective dimensions of the quantity of mass, length and time on the both systems

$u_1 = [M_1^a L_1^b T_1^c]$ and $u_2 = [M_2^a L_2^b T_2^c]$

$$\text{Then} \quad n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$$

By knowing M_1 L_1 T_1, M_2 L_2 T_2 , (a,b,c) and n_1, we can find the value of n_2

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^b$$

Illustration

17) Convert 76 cm of mercury pressure into N m^{-2} using the method of dimensions. (Density of mercury is 13.6 g/cm^3)

Use formula for pressure $P = h\rho g$, here h is height of mercury column, ρ is the density of mercury, g is gravitational acceleration

Solution :

In cgs system, 76 cm of mercury pressure = 76 × 13.6 × 980 dyne cm^{-2}

Let this be P_1. Therefore P_1 = 76 × 13.6 × 980 dyne cm^{-2}

Dimension of pressure is $[\ M^1L^{-1}T^{-2}\ ]$

By using formula

$$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$$

$= 76 \times 13.6 \times 980\ [M_1^1\ L_1^{-1}\ T_1^{-2}] = n_2[M_2^1\ L_2^{-1}\ T_2^{-2}]$

$$n_2 = 76 \times 13.6 \times 980 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^{-1} \left[\frac{T_2}{T_1}\right]^{-2}$$

$$n_2 = 76 \times 13.6 \times 980 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^{-1} \left[\frac{T_2}{T_1}\right]^{-2}$$

Since we want to convert cgs to SI unit we have to convert all units of CGS in terms of SI So mass unit g = 10^{-3}kg, cm = 10^{-2}m time will not change as both units have second as unit

$$n_2 = 76 \times 13.6 \times 980 \left[\frac{10^{-3}\text{kg}}{1\text{kg}}\right]^1 \left[\frac{10^{-2}}{1\text{m}}\right]^{-1} \left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

$$n_2 = 76 \times 13.6 \times 980 \times 10^{-3} \times 10^2 \times 1$$

$$n_2 = 101292.8 \text{ N m}^{-2}$$

$$n_2 = 1.01 \times 10^5 \text{ Nm}^{-2}$$

18) If the units of force, energy and velocity in a new system be 10 N, 5J and 0.5 ms^{-1} respectively, find the units of mass, length and time in that system.

Solution

New system values will be denoted by n_1

From formula

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^b$$

Force

Here n_1 = 10 N , n_2 =1 N and a = 1, b= 1, c = -2 for energy as dimension formula is $[M^1L^1T^{-2}]$

$$1 = 10 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2} \ldots \ldots eq(1)$$

Energy

n_1 = 5J , n_2 = 1J and a = 1, b=2, c = -2 , as for energy dimension formula is $[M^1L^2T^{-2}]$

$$1 = 5\left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2} \ldots\ldots eq(2)$$

N_1 =0.5 m/s , n_2 = 1.0 m/s and a = 0, b = 1, c = -1 , as for velocity dimension formula is $[M^0L^1T^{-2}]$

$$1 = 0.5\left[\frac{M_1}{M_2}\right]^0 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1} \ldots\ldots eq(3)$$

Dividing eq(2) by eq(1) we get

$$1 = 0.5\left[\frac{L_1}{L_2}\right]^1 \; or \; \left[\frac{L_1}{L_2}\right]^1 = 2$$

∴ $L_2 = 0.5\ L_1$

As L_1 =1m L_2 = 0.5m

Substituting value of $\left[\frac{L_1}{L_2}\right]^1 = 2$ in equation 3 we get

$$1 = 0.5\times 2\times \left[\frac{T_1}{T_2}\right]^{-1}$$

$$\left[\frac{T_1}{T_2}\right]^1 = 1$$

As T1 = 1 s , T_2 = 1s

Substituting values of $\left[\frac{L_1}{L_2}\right]^1 = 2 \; and \; \left[\frac{T_1}{T_2}\right]^1 = 1$ in eq(1) we get

$1 = 10\left[\frac{M_1}{M_2}\right]^1 \times 2\times 1$ ∴M_2 = 20 M_1 as M_1 = 1 M_2 = 20 kg

Hence units of mass, length and time are 20 kg, 0.5 m and 1 sec respectively

19) The density of a substance is 8 g/cm^3. Now we have a new system in which unit of length is 5cm and unit of mass 20g. Find the density in this new system

Solution

Let the symbol of unit of length be Ln and mass be Mn.

Since Ln = 5 cm Therefore 1cm = 1/5 Ln, Mn= 20g Therefore 1g = Mn/20

Substituting in formula for density

$$\rho = \frac{m}{V} = \frac{m}{l^3}$$

If volume is 1cm^3 mass is 8gm

$$\rho = \frac{8gm}{1cm^3} = \frac{8\frac{Mn}{20}}{\left(\frac{1}{5}Ln\right)^3}$$

$$\rho = 50\frac{Mn}{(Ln)^3}$$

Thus as per new system density is 50Mn/Ln^3

20) The moment of inertia of body rotating about a given axis is 3.0 kgm^2 in the SI system. What will be the value of moment of inertia in a system of unit of length 5cm and the unit mass is 20g?

Dimension formula for moment of inertia = [ML^2] , thus a = 1 and b = 2

$n_1 = 6$, $M_1 = 1kg$, $L_1 = 1m$, $M_2 = 20g$ and $L_2 = 5cm$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^b$$

$$n_2 = 6\left[\frac{1000g}{20g}\right]^1\left[\frac{100}{5}\right]^2$$

$N_2 = 6\times50\times400 = 120000 = 1.2\times10^5$ units

Exercise 1.03

Convert using dimensional analysis (i) $\frac{18}{5}$ kmph into m s^{-1} (ii) $\frac{5}{18}$ ms^{-1} into kmph (iii) 13.6 g cm^{-3} into kg m^{-3}

Answers:

i) 1 m s^{-1}, ii) 1 kmph, iii) 1.36×10^4 kg m^{-3}

2. Checking the accuracy of various formulae

We can check correctness of the formula based on the principle of homogeneity of dimension. According to this principle, dimensions of various terms of left side of the equation should be same as that of right side

Note that we cannot add or subtract different dimension for example we cannot add L with M. or we cannot add 5metre with 2 kilograms. But we can multiply or divide.

To check the correctness of the given relation we shall write the dimensions of the quantities on both sides of the relation. If the principle of homogeneity is obeyed, the formula is correct.

Note that If equation is dimensionally correct, it is not necessarily that equation describing the relation is correct. However if equation is dimensionally incorrect then equation describing relation is also incorrect.

Illustration

20) Write the dimensions of a and b in the relation, P is pressure, x is distance ad t is time

$$P = \frac{b - x^2}{at}$$

Solution

X^2 is subtracted from b thus b and x^2 must have same dimension, dimension of x^2 = L^2. Therefore dimension of b = $[L^2]$

Expressing given equation in dimensions we get

$$[M^1L^{-1}T^{-2}] = \frac{[L^2]}{a[T]}$$

$$a = \frac{[L^2]}{[M^1L^{-1}T^{-2}][T]}$$

$$a = [M^{-1}L^3T^1]$$

21) Check whether the equation $\lambda = \frac{h}{mv}$ is dimensionally correct

(λ - wavelength, h - Planck's constant, m - mass, v - velocity).

Solution :

Dimension of Planck's constant h is $[ML^2 T^{-1}]$

Dimension of λ is [L]

Dimension of m is [M]

Dimension of v is $[LT^{-1}]$

Representing given equation in dimensions we get

$$[L] = \frac{[ML^1T^{-1}]}{[M][LT^{-1}]}$$

$$[L] = [L]$$

As the dimensions on both sides of the equation are same, the given equation is dimensionally correct

Exercise 1.04

1) Check the correctness of the following equation by dimensional analysis

(i) $F = \frac{mv^2}{r^2}$ where F is force, m is mass, v is velocity and r is radius

(ii) $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ where n is frequency, g is acceleration due to gravity and l is length.

(iii) $\frac{1}{2}mv^2 = mgh^2$ where m is mass, v is velocity, g is acceleration due to gravity and h is height.

2) The number of particle crossing per unit time, normal to cross section is given by

$$N = q\left(\frac{n}{v}\right)$$

Here n is the number of particles per unit volume, v is velocity of particle, find dimension of q

3) Check following formula dimensionally correct or wrong

$$v^2 = \frac{\pi P r^2}{8\eta t}$$

Here v is velocity, P is pressure , η is coefficient of viscosity, t is time

Answers

1 i) wrong, ii) correct, iii) wrong 2) $[L^{-2} T^{-2}]$ 3) Correct

3. Derivation of formulae

Using the principle of homogeneity of dimensions, we can derive the formula of a physical quantity, provided we know the factors on which the physical quantity depends

Using principle of homogeneity of dimensions, equating the powers of M, L, T on both sides of the dimensional equation. Three equations are obtained, on solving the equations we get powers of dimensions. On substituting the values of powers in dimensional equation, we obtain the preliminary form of the relation

Illustration

22) Derive an expression for time period (t) of a simple pendulum, which may depends on mass of bob (m) , length of pendulum(l) and acceleration due to gravity(g)

Solution

Since powers of mass, length and acceleration are not described let us consider a be the power of mass (m), b be the power of l and c be the power of acceleration (g)

Thus we get equation

$$t \propto m^a l^b g^c$$

here a , b and c are the dimensions of m, l and g, now let k be dimension less constant equation becomes $t = k\, m^a l^b g^c$ eq(1)

Writing the dimensions in terms of M, L, T on either side of eq(1) we get

$[M^0L^0T^1] = M^a L^b (LT^{-2})^c$ { dimension of acceleration = $[M^0L^1T^{-2}]$ }

$[M^0L^0T^1] = M^a L^{b+c} T^{-2c}$

Applying the principle of homogeneity of dimensions, we get

a = 0, b+c =0 and -2c = 1

Thus b = ½ Substituting the values in eq(1) we get

$$t = k\ m^0\ l^{1/2}\ g^{-1/2}$$

$$t = k\sqrt{\frac{l}{g}}$$

Value of k a dimension less constant is calculated to be 2π

$$t = 2\pi\sqrt{\frac{l}{g}}$$

23) Obtain by dimensional analysis an expression for the surface tension of a liquid rising in a capillary tube. Assume that the surface tension T depends on mass m of the liquid, pressure P of the liquid and radius r of the capillary tube (Take the constant k = 1 2).

Solution

Let power of m be a, power of pressure be b and radius be c

$T \propto m^a\ p^b\ r^c$

$T = k\ m^a\ p^b\ r^c$

Dimension of Surface tension is T = $[M^1\ L^0\ T^{-2}]$

Dimension of mass m = $[M^1L^0T^{-1}]$

Dimension of Pressure P = $[M^1L^{-1}T^{-2}]$

Dimension of radius r = $[M^0L^1T^0]$

Expressing eq(1) in terms of dimension formula

$[M^1\ L^0\ T^{-2}] = [M^1L^0T^{-1}]^a\ [M^1L^{-1ademy}T^{-2}]^b\ [M^0L^1T^0]^c$

On simplification we get

$[M^1 L^0 T^{-2}] = M^{a+b} L^{-b+c} T^{-a-2b}$

Comparing powers we get equations a+b = 1, -b+c = 0 and -a-2b = -2

∴ b = 1, c =1 and a =0

$T = k P^1 r^1$

As k = ½ given

$$T = \frac{Pr}{2}$$

24) Rotational kinetic energy of the body depends on angular velocity and moment of inertia of the object obtain the formula for Rotational kinetic energy

Where K = Kinetic energy of rotating body and k = dimensionless constant =1

Solution:

$K = kI^a \omega^b$ Dimensions of left side are, $K = [ML^2 T^{-2}]$ Dimensions of right side are, $I^a = [ML^2]^a$, $\omega = [T^{-1}]^b$,

According to principle of homogeneity of dimension, $[ML^2 T^{-2}] = [M^a L^{2a}] [T^{-b}]$
Equating the dimension of both sides, a = 1 and b = 2⇒2 = 2a and –2 = –b

On substitution we get $K = kI\omega^2$, as k =1

$K = I\omega^2$

Exercise 1.05

1) The force F acting on a body moving in a circular path depends on mass m of the body, velocity v and radius r of the circular path. Obtain an expression for the force by dimensional analysis (Take the value of k = 1).

2) When a small sphere moves at low speed through a fluid, the viscous force F opposing the motion, is found experimentally to depend on the radius 'r', the

velocity v of the sphere and the viscosity η of the fluid. Find the force F (take k =6π)

3) To derive the Einstein mass - energy relation, Energy depends on velocity of light and mass, take k =1

4) A gas bubble from an explosion under water oscillates with a period T proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the values of a,b, and c.

5) The depth (x) to which bullet penetrate in a body depends on the coefficient of elasticity (η) and kinetic energy E. Establish the relation between these quantities using the method of dimensions

Answers

1) $F = \frac{mv^2}{r}$ 2) F = 6πηvr 3) $E = mc^2$ 4) a = -5/6 , b = ½, c = 1/3

5) $x \propto (E/\eta)^{1/3}$

1.11.05 Limitation of Dimensional Analysis

1. This method does not give the value of dimensionless constants

2. If a quantity is dependent on more than three factors, having dimension, the formula cannot be derived

3. We cannot obtain formulae containing trigonometrical function, exponential function, log functions etc

4. It does not give information about physical quantity scalar or vector.

5. This method can be used to get exact form of the relation. Which consists more than one part. For example $s = ut + 1/2\ at^2$

1.12 Vernier Calliper

The vernier calliper consists of a main scale and a vernier scale (sliding scale), and enables readings with a precision of 1/200 cm. Figure 1 shows that the main scale is fitted with Jaws C and D on either side, with the straight edges connecting C and D vertically to the main scale forming a right angle. Simultaneously, Jaws E and F are fitted on the vernier scale, which moves over the main scale. When the jaws of the main and the vernier scales contact each other, the zeros of both scales should coincide. If the zeros don't coincide, a zero point calibration must be performed instantly. The distance between C and E or between D and F is the length of the object that is being measured.

We first use an example to demonstrate how to read the vernier caliper, followed by simple equation readings

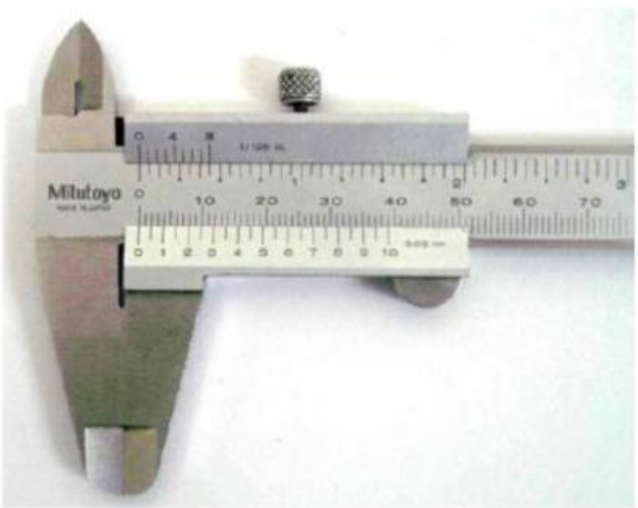


Fig 1.11

The vernier scale (sliding scale) in Figure 1.11 is graduated into 20 divisions sliding scale, which coincide with the 39 smallest divisions on the main scale (i.e., 39 mm). Assuming the length of one division on the vernier scale is S, then S can be obtained as follows:

20S = 39

S = 1.95 mm. (1)

Therefore one division of sliding scale = 2.00 - 1.95 = 0.05 mm

Or least count of vernier = 0.05mm = 0.005 cm

Least count is also known as vernier constant

Illustration

Example 25: The 20 divisions on the vernier scale (sliding scale, VSD) coincide with the 39 smallest markings on the main scale (mm). Find least count

Solution

20 S =39

S = 39/20 = 1.95

Least count = 2 – 1.95 = 0.05 mm

Example 26: Ten divisions on the vernier scale (sliding scale) coincide with 9 smallest divisions on the main scale (mm). Find least count

Solution

10S =9

S= 0.9.

Therefore, least count is equal to 1 - 0.9 = 0.1 mm.

Example 27

In a Vernier 1 cm of the main scale is divided into 20 equal parts. 19 divisions of the main scale coincide with 20 divisions on the vernier scale. Find the least count of the instrument.

Solution
Smallest division on vernier scale =1/20 = 0.05cm = 0.5 mm
19 main scale = 20 Sliding scale(S)
19×0.5 mm = 20 S
S=0.475
Least count= smallest division on vernier scale - vernier scale dive
Least count = 0.5-0.475 = 0.025mm = 0.0025 cm

Example 28) 1 main scale division=0.3 cm . 29 division of main scale coincide with 30 division of vernier scale . Then what will be the least count of vernier
30V.S.D=29M.S.D
The least count of this vernier caliper is......

Smallest division on vernier scale :- 0.3mm

29 main scale divisions = 0.3 * 29 cm

= 3 * 29 mm

= 87mm

= 30 vernier scale divisions

Therefore 1 vernier scale division = 87/30 mm

= 2.9 mm

Least count = 1 main scale div - 1 vernier scale div

= 3mm - 2.9mm

= 0.1mm

Q39) The number of divisions on a vernier scale is 10 and are equal to 9 mm on the main scale. In the measurement of length of a cylinder, the main scale reading is found to be 3.6 cm and the vernier coinciding division was found to be 6. Then length of cylinder is ….?

Solution

10S = 9 mm

S = 0.9 mm

Least count = 1-0.9 = 0.1m

$36 + 6 \times 0.1 = 36.6$ mm = 3.66 cm

1.13 Micrometer screw

Illustration

Q40) A screw gauge of pitch 1mm and 100 division on the circular scale what is its least count

Solution

Dimension, Measurement and error

$$least\ count = \frac{pitch}{no.of\ divison\ on\ circular\ scale}$$

$$least\ count = \frac{1mm}{100} = 0.01mm = 0.001\ cm$$

Exercise 1

Q1) The mass of solid cube is 567 g and each edge has a length of 5 cm. determine the density ρ of the cubic in SI units [Ans 4.536×10^3 kg/m^3]

Q2) Show that $v^2 = u^2 + 2as$ is dimensionally correct, here v is final velocity, u is initial velocity a is acceleration and s is displacement.

Q3) Experiments shows that frequency (n) of tuning fork depends on length (l) of the prong, density(ρ) and the Young's modulus (Y) of its material. On the basis of dimensional analysis obtain formula for frequency . Ans $n = \frac{k}{l}\sqrt{\frac{Y}{\rho}}$

Q4) Round of the following numbers to three significant digits

a) 12.7468 b) 12.75 c) 12.652 $\times10^{12}$ [Ans: 12.7 , 12.8 , 12.6×10^{12}]

Q5) The original length of the wire is (125.6±0.4)cm stretched to (128.8 ±0.2) cm . Calculate the elongation in the wire with error [Ans (3.2±0.6)]

Q6) The measurement of length of rectangles are l = (3.00±0.01)cm and breadth b= (2.00±0.02)cm. What is the area of rectangle? [Ans (6.00±0.08)cm^2]

Q7) The change in the velocity of a body is (15.6 ±0.2) m/s in a time (2.2±0.1)s. Find the average acceleration of the body with error limits

[Ans (7.09 ±0.4) cm/s^2]

Q8) The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. What will be the accuracy in the determination of g . [Ans 2.7%]

Q9) Assuming that the critical velocity of flow of liquid through a narrow tube depends on the radius of the tube, density of the liquid and viscosity of the liquid, then find the expression for critical velocity [Ans $v = \frac{k\eta}{r\rho}$]

Q10) Round off to three significant digit

a) 253674 b) 0.03937 c) 4.065×10^5 [Ans a)254000, b) 0.0394 c) 4.06×10^5]

Q11) The radius of the earth is 6.37×10^6m and its mass is 5.975×10^{24} kg. Find the earth's average density to appropriate significant figure [Ans 5.52×10^3 kh/m^3]

Q12) It is estimated that per minute, each cm^2 of earth receives about 2 calories of heat energy from the sun. This constant is called solar constant. Express solar constant SI units. [Ans 1.kW/m^2]

Hint solar constant $S = \frac{2\ cal}{\min cm^2}$ Convert calories to joule , minutes to second and cm^2 m^2]

Q13) The time period t of the oscillation of a large star, oscillating under its own gravitational attraction, may depends on its mean radius R, its mean density ρ and the gravitational constant. Using dimensional analysis show that t is independent of R and find the formula for t [Ans $t = \frac{k}{\sqrt{\rho G}}$]

Q14) The rotational kinetic energy of a body is given by $E = \frac{1}{2}I\omega^2$, where I is the moment of inertia of the body about its axis of rotation and ω is the angular velocity. Using this relation, obtain the dimensional formula for I. Ans [ML^2]

Q15) Calculate the percentage of error in specific resistance of cylindrical wire, measurements are r = radius of wire = (0.25 ±0.02)cm, Length of wire l = 225±0.1) cm, Resistance of wire = (48±3)Ω Use formula $R = \rho\frac{l}{A}$ [Ans 22.29%]

Q16) A box container of ice-cream having volume 27m^3 is to be made from a cube. What should be the length of side in cm? [Ans 3.0×10^2 cm]

Q17) In two different system of units an acceleration is represented by numbers in ratio 1:2 , whilst a velocity is represented by numbers in ratio 1:3, compare units of length and time [9/2, 2/3]

Q18) In a certain system of absolute units the acceleration produced by gravity in a body falling freely is denoted by 3, the kinetic energy of 100 kg shot moving with 400 metres per second is denoted by 100 and its momentum by 10, find the units of length, time and mass: [Ans mass 200 kg, 40.82 m, 6.12 s]

Solution:

Dimension for gravitational acceleration = L^1T^2

$$3 = 9.8\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2} \ldots . eq(1)$$

Dimension for kinetic energy = $M^1L^2T^{-2}$

$$100 = \left(\frac{1}{2}100\times400^2\right)\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} \ldots . eq(2)$$

Dimension for momentum = $M^1L^1T^{-1}$

$$10 = (100\times400)\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-1} \ldots . eq(3)$$

Squaring eq(3) and dividing by eq(2) we get

$$\frac{10^2}{100} = \frac{(100\times400)^2\left[\frac{M_1}{M_2}\right]^2\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}}{\left(\frac{1}{2}100\times400^2\right)\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}}$$

$$1 = 200\left[\frac{M_1}{M_2}\right]$$

$M_2 = 200\ M_1$

Substituting value of $M_1/M_2 = 1/200$ in eq(2) we get

$$100 = \left(\frac{1}{2}100\times400^2\right)\frac{1}{200}\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} \ldots .eq(4)$$

Dividing eq(4) by eq(1) we get

$$\frac{100}{3} = \frac{\left(\frac{1}{2}100\times400^2\right)\times\frac{1}{200}\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}}{9.8\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2}}$$

$$\frac{100}{3} = \frac{(100\times400^2)\left[\frac{L_1}{L_2}\right]^1}{2\times9.8\times200}$$

$$\frac{100\times9.8\times2\times200}{(100\times400^2)} = \left[\frac{L_1}{L_2}\right]^1$$

$$L_2 = 40.82L_1$$

Dividing eq(3) by eq(1) we get

$$\frac{10}{3} = \frac{(100\times400)\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-1}}{9.8\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2}}$$

$$\frac{10}{3} = \frac{(100\times400)\left[\frac{M_1}{M_2}\right]\left[\frac{T_1}{T_2}\right]^1}{9.8}$$

But $M_1/M_2 = 1/200$

$$\frac{10}{3} = \frac{(100\times400)\left[\frac{T_1}{T_2}\right]^1}{9.8\times200}$$

$$\left[\frac{T_1}{T_2}\right]^1 = \frac{10\times9.8\times200}{3\times100\times400}$$

$$\frac{T_1}{T_2} = \frac{9.8}{60}$$

$$T_2 = (60/9.8)T_1$$

$$T_2 = 6.12 \text{ s}$$

Q19) If the unit of force be the weight of one kilogram, what must be the unit of mass so that the equation F = ma may still be true? [Ans g kg]

Q20) The Young's modulus of steel in CGS system is 19.0×10^{11} dyne cm^{-2}. Express it in the SI system [Ans 19×10^{10} N/m^2]

Q21) We measure the period of oscillation of a simple pendulum. In successive measurements the reading turns out to be 264 s, 256 s, 242 s, 270s and 280 sec for 100 oscillation. If the minimum division in the measuring clock is 1 s, then what will be the reported mean time Calculate the a) absolute error b) relative error c) percentage error d) reported mean time

Ans a) 2, 6,20,12,18 b) 0.04 c) 4 % d) 262±11 s

Q22) A physical quantity z is related to four variable a ,b ,c d, and e are as follows

$$z = \frac{a^2 b^{\frac{1}{3}}}{c^4\, d}$$

Percentage error of measurement in a, b, c, d are 1%, 3%, 3%, 4% respectively. Find the percentage error in z [Ans 19%]

Q23) In the formula x = 2Y Z^2, X and Z have the dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in SI system

[Ans M^{-3} L^{-2} T^8 A^4]

Q24) Finding the dimensions of resistance R and inductance L, speculate what physical quantities (L/R) and $(1/2)LI^2$ represent, where I is current. [Ans time, magnetic energy]

Q25) The parallex of a heavenly body measured from two points diametrically opposite on equator of earth is 30”. If the radius of earth is 6400km, find the distance of the heavenly body from the centre of earth in AU, taking 1AU = 1.5×10^{11} m [Ans 0.586 AU]

Q26) In an experiment, refractive index of glass was observed to be 1.45, 1.5, 1.54, 1.44, 1.54, and 1.53 . Calculate (i) mean value of refractive index (ii) mean absolute error (iii) fractional error (iv) percentage error. Express the result in terms of absolute error and percentage error. [Ans i) 1.51 ii) ±0.04 iii) ±0.03 iv) ±3% ; $\mu = 1.51\pm0.04$; $\mu = 1.51\pm3\%$]

Q27) in a submarine equipped with SONAR, the time delay between generation of probe wave and the reception of its echo after reflection from an enemy submarine is found to be 773.0s. What is the distance of the enemy submarine? [Speed of sound in water = 1450 m/s) [Ans 55825 m]