LAWS OF MOTION

- The law of inertia given by Galileo was represented by Newton as the first law of motion :" If no external force acts on a body, the body at rest remains at rest and a body in motion continues to move with the same velocity." This law gives us definition of force.
- The momentum of a body $\vec{P} = m\,\vec{v}$ is a vector quantity. It gives more information than the velocity. It's unit is $kgms^{-1}$or Ns and dimensional formula $M^1 L^1 T^{-1}$
- Newton's second law of motion : The time-rate of change in momentum of a body is equal to the resultant external force applied on the body and is in the direction of the external force.

$$F = \frac{d\vec{p}}{dt} = m\vec{a}$$

(i) This law gives the value of force.

(ii) The SI unit of force is Newton (N) , 1 N = 1 kg m s^{-2}

(iii) It is consistent with First law

(iv) If F= 0 then acceleration a = 0

- The impulse of force is the product of force and the time for which it acts. when a large force acts for a very small time, it is difficult to measure **F** and Δt but the change in momentum can be measured, which is equal to the impulse of force (**F** Δt)
- Newton's third law of motion: " To every action there is always an equal and opposite reaction."
  Forces always act in pairs, and $\vec{F}_{AB} = -\vec{F}_{BA}$

(i) The action and the reaction act simultaneously.

(ii) They act on different bodies, hence they cannot be cancelled by adding.

(iii) But the resultant of the forces between different parts of the same body becomes zero.

- The law of conservation of momentum is obtained from Newton's second law and the third law. It is written as-"The total momentum of an isolated system remains constant."
- The concurrent forces are those forces of which the lines of action pass through the same point.

  (i) For equilibrium of the body, under the effect of such forces,

  $\sum \mathbf{F} = 0$

  (ii) Moreover, the sum of the corresponding components also should be zero.

  $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$
- Friction is produced due to the contact force between the surfaces in contact. It opposes the impending or the real relative motion.

  Static frictional force $f_s \le f_{s(max)} = \mu_s N$

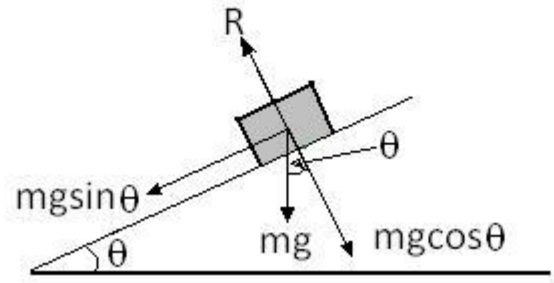
  the kinetic friction is $f_k = \mu_k N$

  $\mu_k < \mu_s$
- The reference frame , in which Newton's first law of motion is obeyed is called the inertial frame of reference and the one in which it is not obeyed is called non-inertial frame of reference. The frame of reference with constant velocity is an inertial frame of reference and one which has acceleration is non-inertial frame of reference.

- The maximum safe speed on level curved road is $v_{max} = \sqrt{\mu_s r g}$

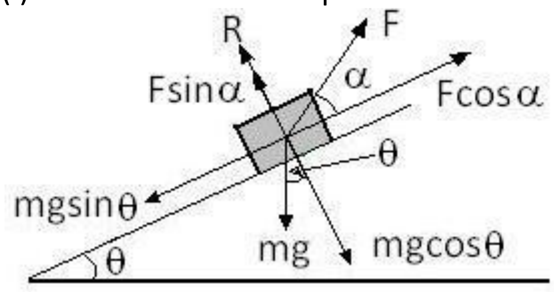
  The maximum safe speed on a banked curved road is

- $v_{max} = \sqrt{rg\left(\frac{\mu_s + tan\theta}{1 - \mu_s tan\theta}\right)}$
- Motion of a body on a friction less inclined plane of inclination θ with the horizontal, its acceleration down the plane is a = gsinθ



- Equilibrium of bodies on rough inclined plane External force F makes an angle of α with inclined plane

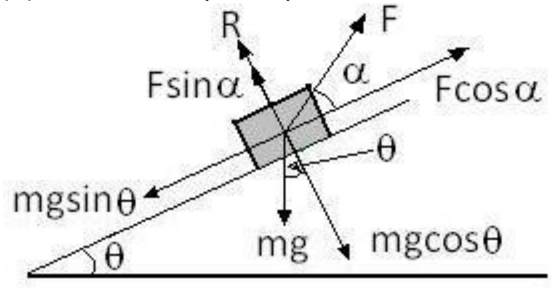
  (i) Motion is down the plane



Fsinα + R = mgcosθ

Fcosα + μR = mgsinθ

(ii) Motion is up the plane

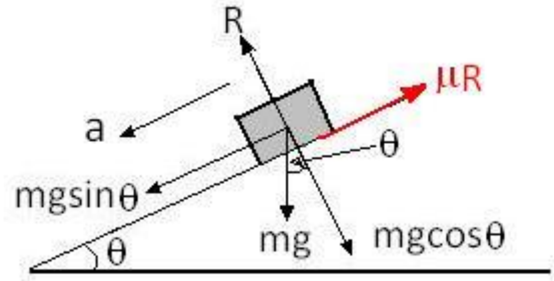


$F\sin\alpha + R = mg\cos\theta$

$F\cos\alpha = \mu R + mg\sin\theta$

- Motion of a body on a Rough inclined plane:

(i) Body moving down : If body moving down with acceleration 'a' then,



$R = mg\cos\theta$

$ma = mg\sin\theta - \mu R$

$ma = mg\sin\theta - \mu mg\cos\theta$

$a = g(\sin\theta - \mu\cos\theta)$

(ii) Body moving up $a = g(\sin\theta + \mu\cos\theta)$

Pseudo Force : In non-inertial frame of reference due to acceleration one more additional force acting on a body in the opposite direction of acceleration of frame of reference is called pseudo force (F_p)

(i) when a man of weight m climbs on the rope with acceleration a then tension in the rope is T = m(g + a).

(ii) When man sliding down with acceleration a then tension in the rope is T = m(g – a).

- When three masses m1, m2, m3 are placed in contact with one another on a smooth horizontal surface and a push F produces an acceleration a in them , then

$$a = \frac{F}{m_1 + m_2 + m_3}$$

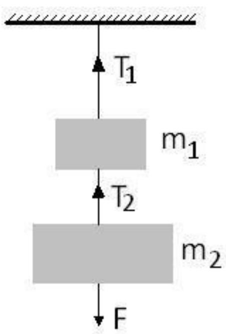
(i) force on m_1 is $F_1 = F$

(ii) force on m_2 is F_2

$$F_2 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

(iii) force on m_3 is F_3

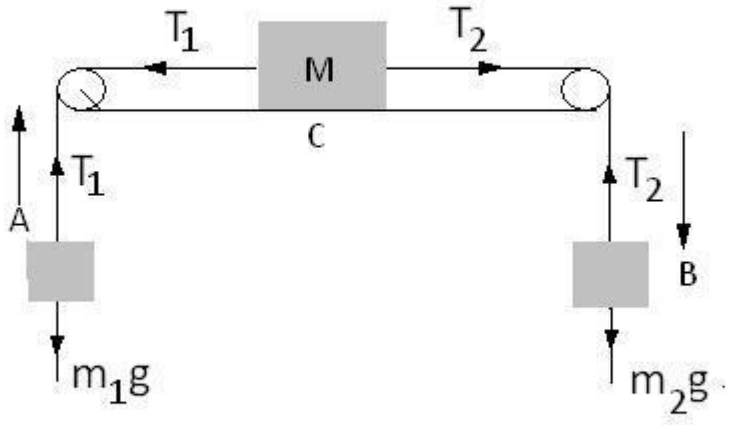
$$F_3 = \frac{m_3 F}{m_1 + m_2 + m_3}$$



- Suppose two masses m1 and m2 are suspended vertically from a rigid support, with the help of strings as shown in figure. When mass m2 is pulled down with force F then

$T_2 = F + m_2g$ and $T_1 = F (m_1 + m_2)g$

- Three bodies on the smooth horizontal table as situation shown in figure



Equation of motion of body A is

$m_1a = T_1 - m_1g$

Equation of motion for body B is

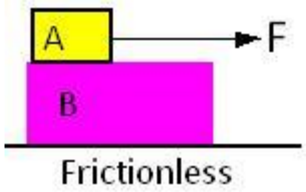
$m_2a = m_2g - T_2$

Equation of motion for body C is

$Ma = T_2 - T_1$

- Motion of two bodies, one resting on other

(A) When a body A of mass m is resting on a body B of mass M and force F is applied on A as shown in figure



(i) When there is no friction between A and B

acceleration of A , $a_A = F/m$

acceleration of B, $a_B = 0$

(ii) When there is a friction between A and B, the body will not slide on B till

$F < f$

Or $F < \mu_s(mg)$

Rather, both A and B will move together with common acceleration

$a_A = a_B = F / (M+m)$

(iii) When F > f, the bodies will move in the direction of applied force, but with different acceleration

Force of dynamic friction f = μk (mg)

This causes the motion of B :

$f = Ma_B$

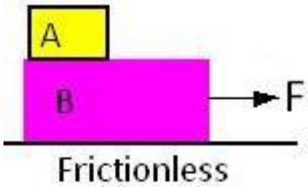
$a_B = f/ M = (\mu_k mg) / M$

For motion of A :

$F - f = ma_A$

$a_A = (F - f) / m$

B) When a force F is applied to the lower body as shown in figure



(i) When there is no friction between A and B

$a_B = F/M$

$a_A = 0$ because there is no pulling force on A relative to B,

(ii) When there is a friction between A and B and two move together,

then $a = F / (m+M)$

in that case, force on A = $F' = ma$

(iii) If $F > \mu_s (m + M)g$,

The bodies A and B will move with different accelerations, such that for body A:

$ma_A = \mu_k mg$

$aA = \mu kg$

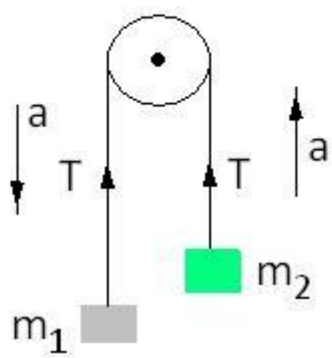
for body B:

F - f = MaB

aB = (F -f)/ M

- Pulley

Case I



Here $m_1 > m_2$

For m_1 :

$m_1a = m_1g - T$

For m_2 :

$m_2a = T - m_2g$

Solving we get

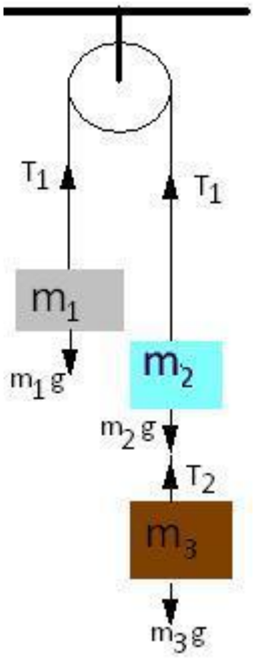
acceleration

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

tension

$$T = \frac{(2m_1m_2)g}{m_1 + m_2}$$

- Case II



For m1 :

$m_1 a = T_1 - m_1 g$ --eq(1)

For m_2 :

$m_2 a = T_2 + m_2 g - T_1$

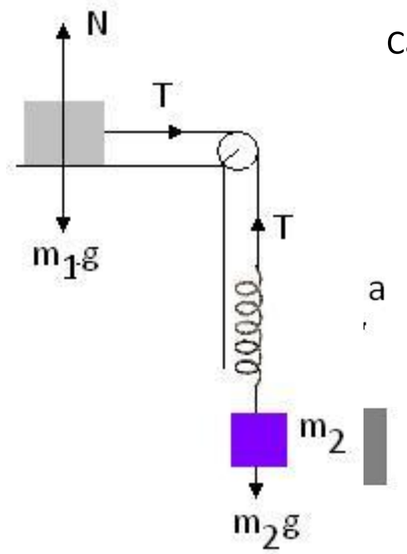
for mass m_3 :

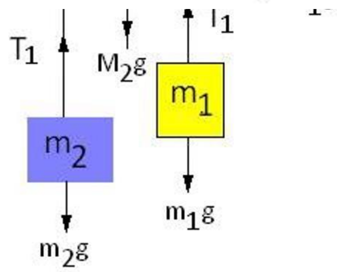
$m_3 a = m_3 g - T_2$ --eq(3)

On solving we get acceleration

On substituting value of a in equation (1) and (2) we can find tension T_1 and T_2

Case III





Equation of Mass M1:

M1a = M1g - T --eq(1)

For M_2

$M_2a = T - (2T_1+M_2g)$

For m_1

$m_1a' = (T_1 - m_1g) - m_1a$

For m2

$m_2a' = m_2g + m_2a - T$

where a' is the acceleration of masses m1 and m2 solving these equations we can get value of a'

Case IV

For mass m_1

$m_1a = T$

For mass m_2

$m_2a = m_2g - T$

On solving acceleration

$$a = \frac{m_2 g}{m_1 + m_2}$$

Tension T

$$T = \frac{m_1 m_2 g}{m_1 + m_2}$$

If x is the extension in spring then T = kx (Here k is spring constant)

or x = T/k

$$x = \frac{m_1 m_2 g}{k(m_1 + m_2)}$$

WORK ENERGY AND POWER

- The product of the magnitude of the displacement during the action of a force and the magnitude of the component of the force in the direction of displacement is known as work. Its unit in joule and its dimensional formula is $M^1 L^2 T^{-2}$
- Work = FS cos θ

  Here θ is the angle between displacement (S) and direction of force(F)

  (i) If θ = 0 then W = Fs

  (ii) If θ = π/2 then W = 0

  (iii) If θ = π then W = -FS

  (iv) Work done against friction on horizontal surface is :

  W = μR(S) = μmg(S)

  (v) Work done against friction while moving up an inclined plane is :

  W = μmgcosθ(S)
- Work done by variable force is given by

$$W = \int_i^f \vec{F} \cdot \overrightarrow{dS}$$

  (i) If a variable force and displacement due to it, are in the same direction the area enclosed by Force -displacement curve gives value of work.
- The ability of a body to do work by virtue of its motion is known as its Kinetic energy. It is a scalar quantity. If the velocity of a body of mass `m' is `v' its kinetic energy is

$$K = \frac{1}{2} m\, v^2 = \frac{p^2}{2m}$$

- Work Energy Theorem : Work done by a resultant force on a body is equal to the change in its kinetic energy

- When a body has the ability to do work due to its position in a force field or its configuration. It is known as potential energy. It is scalar quantity.
- If the gravitational potential energy, due to the gravitational field of Earth, is randomly taken to be zero on its surface, the potential energy of a body of mass m, at height h is mgh, where g is the gravitational acceleration. The value of `h' is negligible compared to the radius of the earth.
- The sum of the potential energy (U) and the kinetic energy (K) of a substance is called the mechanical energy. E = K + U.
- Considering potential energy of a spring as zero in its normal state, if its length is changed by x, the potential energy of the spring is

$$U = \frac{1}{2}kx^2$$

- Here k is the spring constant. Unit of k is N/m and dimensional formula is $M^1 L^0 T^{-2}$
- The forces for which work done is independent of the path of motion of the body but depends only on initial and final positions, are called conservative force. The force of gravitation or the restoring force developed in a spring due to its compression or extensions are conservative forces.
- The relation between the conservative force and the potential energy is

$$F = -\frac{dU}{dx}$$

- The time-rate of doing work is called power. Its unit is watt (J/s). Its dimensional formula is $M^1 L^2 T^{-3}$

  The power P = W / t or $P = \vec{F}.\vec{v}$
- 1 horse power = 746 watt Unit of electric energy for domestic use is

  1 unit = 1 kWh = 3.6×10^6 J
- Collision

  (i) Suppose a body dropped from a height ho above the ground strikes the ground with velocity v_o. Let the body rebound with a velocity v_1 and go to height h_1, then coefficient of restitution

$$e = \frac{v_1}{v_0} = \sqrt{\frac{2gh_1}{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$

If v_n is velocity with which the body rebounds after n collision to height h_n then

$$e^n = \frac{v_n}{v_0} = \sqrt{\frac{h_n}{h_0}}$$

(ii) A body of mass m_1 moving with velocity u_1 collides with a body of mass m_2 moving with velocity u_2

If final velocities of m_1 is v_1 and final velocities of m_2 is v_2

$$e = -\frac{v_1 - v_2}{u_1 - u_2}$$

(iii) Value of 'e' depends on the types of materials of bodies colliding

(iv) If during collision of two bodies the kinetic energy is conserved the collision said to be elastic. and value of e = 1

(v) In case of complete inelastic collision, bodies colliding move together after collision with a common velocity v. For perfectly inelastic collision e = 0

(vi) When the collision is not perfectly elastic, then the general expression for velocities after direct impact are

$$v_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} + \frac{(1+e)m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{(1+e)m_1u_1}{m_1 + m_2} + \frac{(m_2 - em_1)u_2}{m_1 + m_2}$$

- Spring

  (i) Spring constant k = F/x = restoring force per unit extension

(ii) If a spring of spring constant k is cut into two equal parts, then the spring constant of each part = 2k

in general, when the spring is cut into n parts of equal length, spring constant of each part = nk

(iii) When a spring of spring constant k is cut into to parts of unequal lengths l1 and l2 the spring constants of the two parts are

$$k_1 = \frac{k(l_1 + l_2)}{l_1}$$

$$k_2 = \frac{k(l_1 + l_2)}{l_2}$$

(iv) When two springs of spring constant k_1 and k_2 are joined in series, and a mass is attached to the free end, the equivalent spring constant k of the combination is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

(v) When mass is attached between to springs of spring constant k_1 and k_2 are joined in parallel, the equivalent spring constant k is given by

$k = k_1 + k_2$

State of equilibrium

A) Stable equilibrium

(i) When displaced from equilibrium position, particle tends to come back.

(ii) Potential energy is minimum

(iii) $\frac{d^2U}{dx^2} = positive$

(iv) $F = -\frac{dU}{dx} = 0$

B) Unstable equilibrium

(i) When displaced from equilibrium position, particle tends to move away from equilibrium position.

(ii) Potential energy is maximum

(iii) $\frac{d^2U}{dx^2} = negative$

(iv) $F = -\frac{dU}{dx} = 0$

C) Neutral equilibrium

(i) Particle always remains in the state of equilibrium irrespective of any displacement

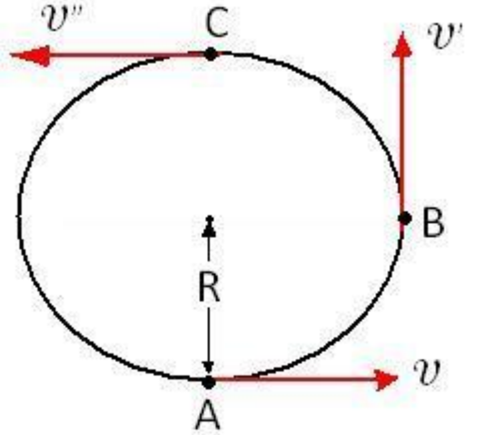
(ii) Potential energy is constant

(iii) $\frac{d^2U}{dx^2} = 0$

(iv)) $F = -\frac{dU}{dx} = 0$

- Velocity at lowest point A such that bob of mass m tied to the end of light string can just reach the point C the highest position $V = \sqrt{5gR}$

  And velocity at point B , $V' = \sqrt{5gR}$Then velocity at C is v" = 0



GRAVITATION

- Gravitational constant G value =6.67 × 10^{-11} Nm^2 kg^{-2}
- If ρ is the density of object then gravitational acceleration g

$$g = \frac{4}{3}\pi G R \rho$$

- Acceleration due to Gravity

(i) Surface

$$g = \frac{GM}{R_e^2}$$

(ii) At a height h from surface of earth

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

If h<<R

$$g' = g\left(1 - \frac{2h}{R}\right)$$

(iii) At a depth d form the surface of earth

$$g' = \frac{g}{\left(1 - \frac{d}{R}\right)}$$

(iv) Effect of rotation of earth at latitude λ

$g' = g - R\omega^2\cos 2\lambda$

(i) at equator $\lambda = 0$, $g' = g - R\omega^2$ (minimum)

(ii) At the pole $\lambda = 90°$, $g' = g$ (maximum)

(iii) If the rate of rotation of earth increases, the value of acceleration due to gravity decreases at all places on the surface of earth except at pole

(iv) If the earth starts rotating with an angular velocity seventeen times its present speed of rotation, the object lying on equator will fly off the equator because the value of g will then be zero at equator. In this situation the length of day will be 1.4 hr.

- Field Strength

(i) Gravitational field strength at a point in gravitational field is defined as gravitational force per unit mass

$$\vec{E} = \frac{\vec{F}}{M}$$

(ii) Due to point mass

$$E = \frac{GM}{R^2}$$

(iii) Due to solid sphere and spherical shell

(a) inside point

$$E = \frac{GM}{R^2}r$$

(b) at centre E = 0

(c) on the surface

$$E = \frac{GM}{R^2}$$

(d) out side points

$$E = \frac{GM}{r^2}$$

r → ∞ ; E = 0

(iv) ring

(a) on the axis of a ring

$$E_r = \frac{GM\, r}{(R^2 + r^2)^{\frac{3}{2}}}$$

(b) At centre of ring E = 0

(c) If r > > R

$$E = \frac{GM}{r^2}$$

(v) At a point on the axis at a distance r from centre of disc of radius R and mass per unit area ρ

$$E_r = 2\pi\rho G\left[1 - \frac{r}{\sqrt{R^2 + r^2}}\right]$$

- Gravitational potential

(i) Solid sphere

(a) point inside of sphere

$$V = -\frac{GM}{2R^3}(3R^2 - r^2)$$

(b) At surface of sphere

$$V = -\frac{GM}{R}$$

(c) At centre of sphere

$$V = -\frac{3}{2}\frac{GM}{R}$$

(ii) Due to spherical shell

(a) inside and outside the shell

$$V = -\frac{GM}{R}$$

(iii) On axis of ring

$$V = -\frac{GM}{\sqrt{R^2 + r^2}}$$

(a) At centre of ring

$$V = -\frac{GM}{R}$$

- Orbital velocity of satellite at height h above the surface of earth

$$v_0 = \sqrt{\frac{GM}{(R+h)}}$$

$$v_0 = \sqrt{\frac{gR^2}{(R+h)}}$$

(i) velocity very near to surface of earth

$$v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

(a) If the gravitational force is inversely proportional to n^{th} power of distance r, then the orbital velocity of satellite, $v_0 = \frac{r^n}{2}$

(b) and time period is

$$T \propto r^{\frac{n+2}{2}}$$

(ii) The orbital velocity of satellite is independent of mass of satellite but depends upon the mass and radius of the planet around which the rotation is taking place

(iii) Value of orbital velocity for satellite, near the surface of earth is 7.92 km/s

(iv) Value of time period for satellite, near the surface of earth is 84.6 minutes

(v) The angular velocity of a satellite orbiting close to the surface of earth is = 0.001237 rad/s

(iv) The orbital velocity of a satellite decreases with an increase in the radius of orbit

(v) For stationary satellite the orbital velocity increases with increase in radius of orbit

(vi) Orbital velocity of Geostationary satellite is 3.08 km/s and its relative angular velocity with respect to earth is zero

- Total energy of a satellite is the sum of kinetic energy E and potential energy U

  Kinetic energy of satellite K

$$K = \frac{GMm}{2r}$$

Potential energy of satellite U

$$U = -\frac{GMm}{r}$$

Total energy of satellite = E+U

$$E_T = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E_T = -\frac{GMm}{2r}$$

$$E_T = -K = \frac{1}{2}U$$

here negative sign indicates that the satellite is bound to the surface of earth.

- Binding velocity and escape velocity

  (i) Binding energy of satellite

$$E_B = \frac{GMm}{2r}$$

  (ii) Value of escape velocity

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

  Escape velocity for earth is 11.2 km/s

- Relation between orbital speed and escape velocity

  Escape velocity (v_e) = √2 × orbital velocity(v_o)
- When a body is projected horizontally with velocity v, from any height from the surface of earth, then the following possibilities are there.

  (i) If $v < v_o$, the body fails to revolve around the earth and finally falls to the surface of earth.

  (ii) If $v = v_o$, the body will revolve around the earth in circular orbit.

  (iii) If $v < v_e$ the body will revolve around the earth in elliptical orbit.

  (iv) If $v = v_e$, the body will escape from the gravitational field of earth.

  (v) If $v > v_e$ the body will escape, following a hyperbolic path. and the velocity of body v', moving in interstellar space is v'

$$v' = \sqrt{v^2 - v_e^2}$$