

# RAY OPTICS AND OPTICAL INSTRUMENTS

## SECTION I

### REFLECTION OF LIGHT

#### Nature of light

- Light is an electromagnetic radiation which causes sensation in eyes.
- Wavelength of visible light is 400 nm to 750 nm
- Speed of light in vacuum is highest speed attainable in nature  $3.0 \times 10^8$  m/s
- Wavelength of light is very small compared to the size of the ordinary objects, thus light wave is considered to travel from one point to another along straight line joining two points.
- The straight path joining two points is called ray of light.
- Bundle of rays is called a beam of light
- Light show optical phenomenon such as reflection , refraction, interference and diffraction

#### REFLECTION OF LIGHT BY SPHERICAL MIRRORS

**Law of reflection.** i) The angle of incidence (angle between incident ray and normal to the surface) and angle of reflection (angle between reflected ray and normal to the surface) are equal.

ii) Incident ray, reflected ray and normal to the reflecting surface at the point of incidence lie in the same plane.

Note : Normal to the curved surface always passes through the centre of curvature

**Sign convention:** (i) All distances are measured from the pole of the mirror.

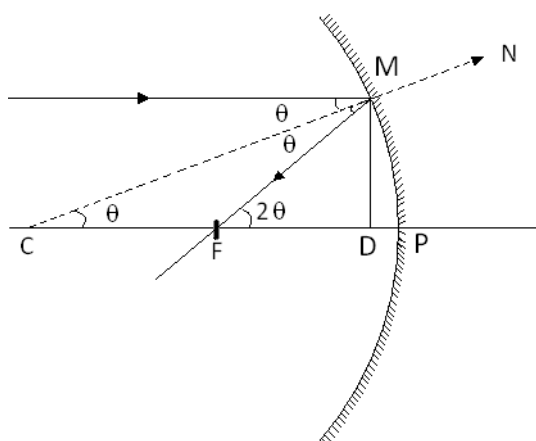
(ii) The distance in the direction of incidence light is taken as positive.

(iii) Distance in opposite to the direction of incident light is taken as negative

(iv) Distance above the principal axis is taken as positive.

(v) Distance below the principal axis taken as negative.

#### FOCAL LENGTH OF SPHERICAL MIRROR



Let P the pole of concave mirror, F be focal point and C be the centre of curvature.

Consider incident light parallel to principal axis strikes the mirror at point M and reflected rays pass through focal point F.

MD is perpendicular from M on principal axis.

Let  $\angle MCP = \theta$

Then from geometry of figure,  $\angle MFP = 2\theta$ .

$$\text{Now } \tan \theta = \frac{MD}{CD} \text{ and } \tan 2\theta = \frac{MD}{FD} \text{ -eq(1)}$$

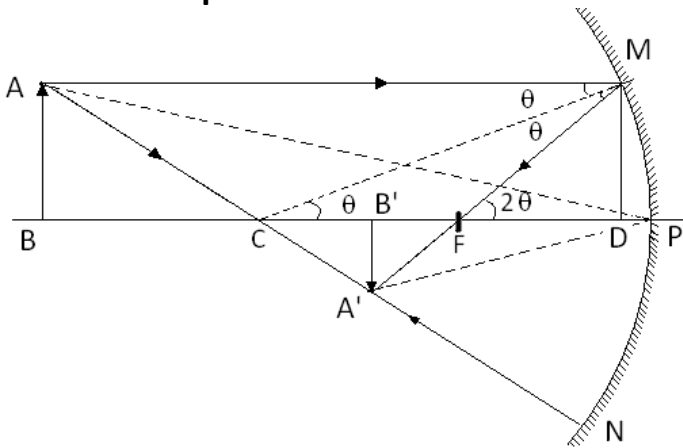
For small  $\theta$ ,  $\tan \theta = \theta$  and  $\tan 2\theta = 2\theta$

Therefore from eq(1)  $2 \frac{MD}{CD} = \frac{MD}{FD}$

Thus  $FD = CD/2$  But  $CD = R$  and  $FD = f$

Thus  $f = R/2$

**The Mirror equation**



As shown in figure AB is object while A'B' is image of the object. AM and AN are two incident rays emitted from point A. Let F be focal point and  $FD = f$  focal length

For small aperture, DP will be very small neglecting DP we will take  $FD = FP = f$

i) In  $\Delta A'B'F$  and  $\Delta MDF$  are similar as  $\angle MDF = \angle A'B'F$

$$\frac{A'B'}{MD} = \frac{B'F}{FD}$$

As  $MD = AB$  and  $FD = FP$

$$\frac{A'B'}{AB} = \frac{B'F}{FP}$$

ii) In  $\Delta A'B'P$  and  $\Delta ABP$  are also similar. Therefore

$$\frac{A'B'}{AB} = \frac{PB'}{PB}$$

Thus comparing above two equations

$$\frac{B'F}{FP} = \frac{PB'}{PB}$$

But  $B'F = PB' - PF$

$$\frac{PB' - PF}{FP} = \frac{PB'}{PB} \quad - eq(1)$$

From sign convention

Focal length =  $PF = -f$

Image distance  $PB' = -v$

Object distance =  $PB = -u$

Putting the values in equation (1)

$$\frac{-v - (-f)}{-f} = \frac{-v}{-u}$$

$$\frac{v - f}{f} = \frac{v}{u}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

This relation is known as mirror equation

### Magnification

In  $\Delta A'B'P$  and  $\Delta ABP$  are also similar. Therefore

$$\frac{A'B'}{AB} = \frac{PB'}{PB}$$

From sign convention

PB' = image distance = -v

PB = object distance = -u

A'B' = size of image = -h'

AB = size of object = h

$$\frac{-h'}{h} = \frac{-v}{-u}$$

$$\frac{h'}{h} = -\frac{v}{u}$$

Magnification,  $m = \text{size of the image} / \text{size of object} = h'/h$

$$m = -\frac{v}{u}$$

Note : If  $m$  is negative , image is real and inverted

If  $m$  is positive image is virtual and inverted

If  $|m| = 1$  , size of the object = size of image

If  $|m| > 1$  size of image > size of the object

If  $|m| < 1$  size of the image < size of the object

### Solved Numerical

Q) An object is placed in front of concave mirror at a distance of 7.5 cm from it. If the real image is formed at a distance of 30 cm from the mirror, find the focal length of the mirror.

What should be the focal length if the image is virtual?

Solution: Case I: When the image is real

U = -7.5 cm; v = -30cm; f = ?

We know that

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-30} + \frac{1}{-7.5}$$

$$\frac{1}{f} = \frac{-5}{30}$$

$$f = -6 \text{ cm}$$

The negative sign shows that the spherical mirror is convergent or concave

Case II : When image is virtual

$$u = -7.5 \text{ cm} ; v = +30 \text{ cm}$$

We know

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{-7.5}$$

$$\frac{1}{f} = \frac{-3}{30} = \frac{-1}{10}$$

$$f = -10 \text{ cm}$$

Q) An object 0.5 cm high is placed 30 cm from convex mirror whose focal length is 20 cm. Find the position, size and nature of the image.

Solution: We have

$$U = -30 \text{ cm}, f = +20 \text{ cm}$$

Form mirror formula

$$\frac{1}{20} = \frac{1}{v} + \frac{1}{-30}$$

$$\frac{1}{v} = \frac{1}{12}$$

$$V = 12 \text{ cm}$$

The image is formed 12 cm behind the mirror. It is virtual and erect

$$m = h'/h = -v/u = -12/30$$

$$m = \frac{h'}{h} = \frac{-v}{u} = \frac{-12}{-30} = 0.4$$

$$h' = mh = 0.4 \times 0.5 = 0.2 \text{ cm}$$

positive sign of m indicate image is erect.

Q) A thin rod AB of length 10 cm is placed on the principal axis of a concave mirror such that its end B is at a distance of 40 cm from the mirror and end A is further away from the mirror. If the focal length of the mirror is 20cm, find the length of the image of rod

Solution: given  $f = -20 \text{ cm}$ , distance of B =  $u_1 = -40 \text{ cm}$ ,

Since B is at centre of curvature image will be formed at  $-40 \text{ cm}$

Distance of A  $u_2 = -50 \text{ cm}$

$$\frac{1}{-20} = \frac{1}{v} + \frac{1}{-50}$$

$$V = -33.3 \text{ cm}$$

Image of A is also on the side of object , Now length of image =  $40 - 33.3 = 6.70 \text{ cm}$

## SECTION II

# REFRACTION OF LIGHT

### REFRACTION

When an obliquely incident ray of light travels from one transparent medium to other it changes its direction of propagation at the surface separating two media. This phenomenon is called as refraction. This phenomenon is observed as light has different velocities in different media.

Laws of refraction

- (i) The incident ray, refracted ray and the normal to the interface at the point of incidence, all lie in the same plane
- (ii) The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant.

$$\frac{\sin i}{\sin r} = n_{21}$$

Where  $n_{21}$  is constant, called refractive index of the second medium with respect to first medium. Equation is known as Snell's law of refraction

Case i)  $n_{21} > 1$ ,  $r < i$ , refracted ray bends towards normal, then medium 2 is called optically denser medium

Case ii)  $n_{21} < 1$ ,  $r > i$ , refracted ray bends away from normal, then medium 2 is called optically rarer medium

If  $n_{21}$  is refractive index of medium 2 with respect to medium 1 and  $n_{12}$  is the refractive index of medium 1 with respect to medium 2 then

$$n_{21} = \frac{1}{n_{12}}$$

Similarly

$$n_{32} = n_{31} \times n_{12}$$

### General form of Snell's law

$$n_{21} = \frac{v_1}{v_2}$$

Absolute refractive index is given by formula

$$n = \frac{c}{v}$$

Thus  $n_1 = c/v_1$  and  $n_2 = c/v_2$

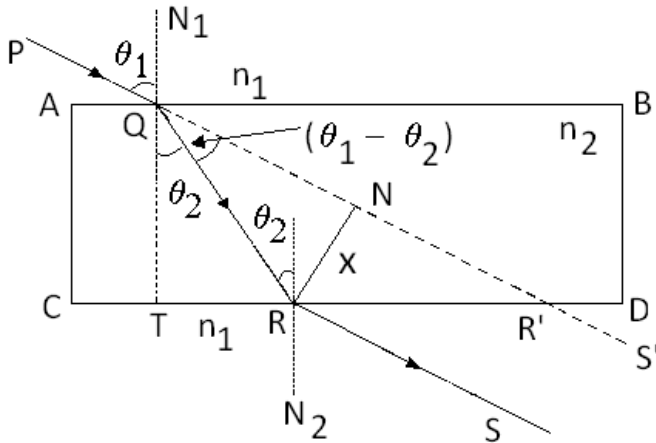
$v_1 = c/n_1$  and  $v_2 = c/n_2$

$$n_{21} = \frac{C/n_1}{C/n_2} = \frac{n_2}{n_1}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

**Lateral shift**



As shown in figure light ray undergoes refraction twice, once at top (AB) and then from bottom (CD) surfaces of given homogeneous medium.

The emergent ray is parallel to PQR'S' ray. Here PQR'S' is the path of light in absence of the other medium.

Since emergent ray is parallel to the incident ray but shifted sideways by distance RN. This RN distance is called lateral shift (x)

**Calculation of lateral shift**

Let  $n_1$  and  $n_2$  be the refractive indices of the rarer and denser medium, respectively. Also  $n_1 < n_2$  From figure  $\angle RQN = (\theta_1 - \theta_2)$ ,  $RN = X$

i) From  $\Delta QRN$ ,  $\sin(\theta_1 - \theta_2) = RN/QR = X/QR$  --eq(1)

ii) In  $\Delta QTR$ ,  $\cos \theta_2 = QT/QR$   
 $\therefore QR = QT/\cos \theta_2$  ;  $QR = t/\cos \theta_2$

From equation (1)

$$\sin(\theta_1 - \theta_2) = \frac{x}{\frac{t}{\cos \theta_2}}$$

$$x = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2}$$

If angle of incidence  $\theta_1$  is every small,  $\theta_2$  will also be small  
 $\sin(\theta_1 - \theta_2) \approx (\theta_1 - \theta_2)$  and  $\cos \theta_2 \approx 1$

$$x = t(\theta_1 - \theta_2)$$

$$x = t\theta_1 \left(1 - \frac{\theta_2}{\theta_1}\right)$$

From Snell's law

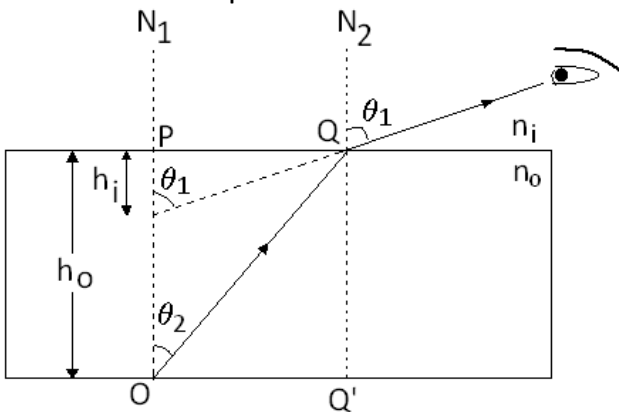
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\theta_1}{\theta_2} = \frac{n_2}{n_1}$$

Thus

$$x = t\theta_1 \left(1 - \frac{n_1}{n_2}\right)$$

**Real Depth and Virtual depth**

Is Another example of lateral shift.



$n_i$  is the refractive index of observer medium,  $n_o$  is the refractive index of object medium.  $h_o$  is the real depth and  $h_i$  is virtual depth.

Applying Snell's law at point Q,  
 $n_i \sin\theta_1 = n_o \sin\theta_2$  For normal incidence  $\theta_1$  and  $\theta_2$  are very small.

$$\therefore \sin\theta \approx \theta \approx \tan\theta$$

$$n_i \tan\theta_1 = n_o \tan\theta_2 \dots \text{Eq(1) But from}$$

figure

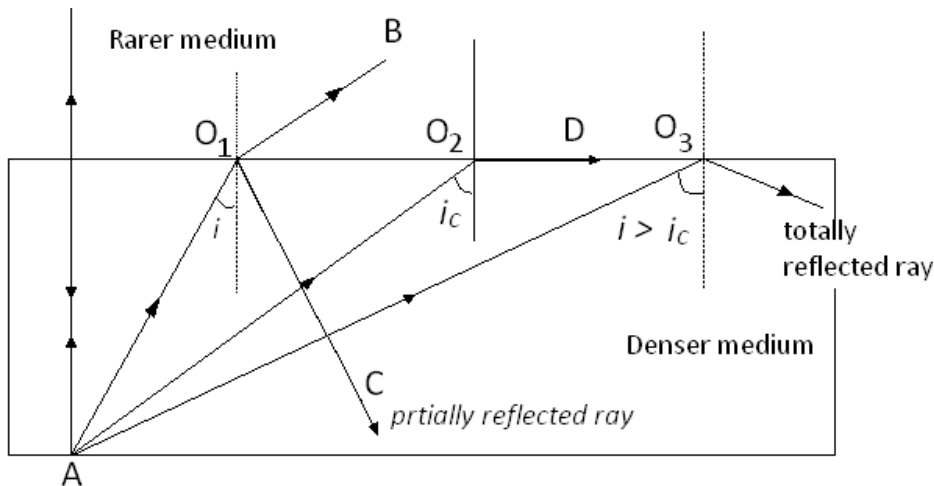
$$\tan\theta_1 = \frac{PQ}{PI} = \frac{PQ}{h_i} \text{ and } \tan\theta_2 = \frac{PQ}{PO} = \frac{PQ}{h_o}$$

Thus from equation (1)

$$n_i \frac{PQ}{h_i} = n_o \frac{PQ}{h_o}$$

$$\frac{n_i}{n_o} = \frac{h_i}{h_o}$$

**Total Internal Reflection**



When light travels from optically denser medium to rarer medium at the interface, it is partly reflected back into same medium and partly refracted to the second medium. This reflection is called the *internal reflection*.

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal.

for example. As shown in figure , The incident ray  $AO_1$  is partially reflected ( $O_1 C$ ) and partially transmitted ( $O_1 B$ ) or refracted.

the angle of refraction ( $r$ ) being larger than the angle of incidence ( $i$ ).

As the angle of incidence increases, so does the angle of refraction, till for the ray  $AO_2$  the angle of refraction is  $\pi/2$ . The refracted ray is bent so much away from the normal that it grazes the surface at the interface between the two media.

This is shown by the ray  $AO_2 D$  in Fig.

If the angle of incidence is increased still further (e.g, the ray  $AO_3$ ), refraction is not possible. And the incident ray is totally reflected. This is called total internal reflection.

The angle of incidence for which angle of refraction is  $\pi/2$  is called critical angle

From Snell's law  $\sin i / \sin r = n_2 / n_1$  If  $n_2$  is air then  $n_1 = 1$

When  $i = i_c$  ( critical angle ) then  $r = \pi/2$ . Let  $n_1$  refractive index of denser medium =  $n$  Then

$$\sin i_c = 1/n$$

## Solved Numerical

Q) A ray of light is incident at angle of  $60^\circ$  on the face of a rectangular glass slab of the thickness 0.1m and refractive index 1.5 . Calculate the lateral shift

Solution: Here  $i = 60^\circ$  ;  $n = 1.5$  and  $t = 0.1$  m

From Snell's law



$$n = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{n} = \frac{\sin 60}{1.5} = \frac{0.866}{1.5} = 0.5773$$

Thus  $r = 35^{\circ}15'$

Now lateral shift

$$x = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2} = \frac{0.1 \times \sin(60^{\circ} - 35^{\circ}15')}{\cos 35^{\circ}15'} = 0.0512 \text{ m}$$

Q) A tank filled with water to a height of 12.5cm. The apparent depth of needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Solution:

Case I : When the tank is filled with water:

Real depth  $h_o = 12.5$  cm Apparent depth  $h_i = 9.4$  cm,  $n_i = \text{air} = 1$

From formula

$$\frac{n_i}{n_o} = \frac{h_i}{h_o}$$

$$\frac{1}{n_o} = \frac{9.4}{12.5}$$

$$n_o = 1.33$$

Case II: When tank filled with the liquid of refractive index =  $n_o = 1.63$

$$\frac{1}{1.63} = \frac{h_i}{12.5}$$

$$h_i = \frac{12.5}{1.63} = 7.67 \text{ cm}$$

Therefore the distance through which the microscope to be moved =  $9.4 - 7.67 = 1.73$  cm

Q) A fish rising vertically to the surface of water in a lake uniformly at the rate of 3 m/s observes kingfisher bird diving vertically towards water at the rate 9 m/s vertically above it. If the refractive index of water is  $4/3$ , find the actual velocity of the dive of the board.

Solution: Velocity of bird with respect to stationary fish in water = 6m/s

Thus apparent displacement of bird in 1 sec  $h_i = 6$  m

$n_i = 4/3$ ,  $n_o = 1$  ( air)

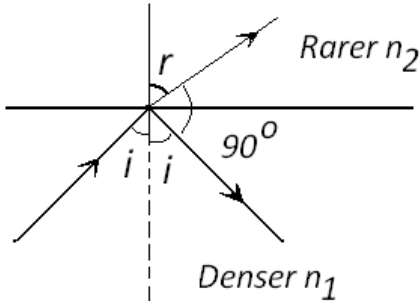
Now from formula

$$\frac{n_i}{n_o} = \frac{h_i}{h_o}$$

$$\frac{4}{3} = \frac{6}{h_o}$$

$$h_o = \frac{6 \times 3}{4} = 4.5 \text{ m/s}$$

Q) A ray of light from a denser medium strikes a rarer medium at an angle of incidence  $i$ . If the reflected and the refracted rays are mutually perpendicular to each other, what is the value of the critical angle?



Solution: From Snell's law, we have

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

According to given problem

$$i+r+90 = 180$$

$$R = 90-i$$

Thus from above equation

$$n_{21} = \frac{\sin(90 - i)}{\sin i} = \cot i$$

By definition for critical angle

$$i_c = \sin^{-1} \left( \frac{1}{n_{21}} \right)$$

$$i_c = \sin^{-1} \left( \frac{1}{\cot i} \right)$$

$$i_c = \sin^{-1}(\tan i)$$

### Examples of total internal reflection

(i) **Mirage:** In summer, air near the surface becomes hotter than the layer above it. Thus as we go away from the surface air becomes optically denser.

Ray of light travelling from the top of tree or building towards the ground it passes through denser air layer to rarer layers of air, as a result refracted rays bend away from normal and angle of incidence on consecutive layers goes on increasing, at a particular layer angle of incidence is more than critical angle rays gets internally reflected and enters the eye of observer and observer observes inverted image of the object. Such inverted image of distant tall object causes an optical illusion to observer. This phenomenon is called mirage.

(ii) **Diamond:** Brilliance of diamond is due the internal reflection of light. Critical angle of diamond- air interface is ( $24.4^\circ$ ) is very small, it is very likely to undergo total internal reflection inside it. By cutting the diamond suitably, multiple total internal reflections can be made to occur.

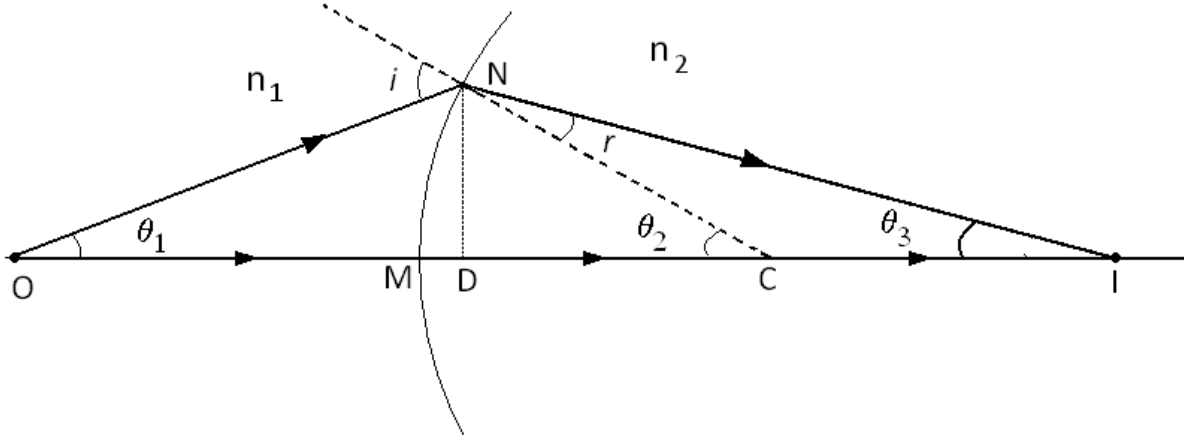
(iii) **Prism:** Prism designed to bend light by  $90^\circ$  or  $180^\circ$  make use of total internal reflection. Such a prism is also used to invert images without changing the size.

(iv) **Optical fibres.** Optical fibres make use of the phenomenon of total internal reflection. When signal in the form of light is directed at one end of the fibre at suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out of the other end.

Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher

than that of the cladding. The main requirement in fabrication of optical fibres is that there should be very little absorption of light as it travels for long distances inside them. This has been achieved by purification and special preparation of materials such as quartz. In silica fibres, it is possible to transmit more than 95% of light over a fibre length of 1km

**Refraction at spherical surface and by lenses**



MO is object distance = - u

MC is radius of curvature = +R

MI is image distance = +v

The ray incident from a medium of refractive index  $n_1$ , to another of refractive index  $n_2$ . We take the aperture of the surface so small that we can neglect the distance MD.

$$\tan\theta_1 = \frac{ND}{OD} = \frac{MN}{OM}$$

$$\tan\theta_2 = \frac{ND}{CD} = \frac{MN}{CM}$$

$$\tan\theta_3 = \frac{ND}{ID} = \frac{MN}{IM}$$

$\theta_1, \theta_2, \theta_3$  are very small, for small  $\theta$ ,  $\tan \theta = \theta$

$$\theta_1 = \frac{ND}{OD} = \frac{MN}{OM} = \frac{MN}{-u}$$

$$\theta_2 = \frac{ND}{CD} = \frac{MN}{CM} = \frac{MN}{R}$$

$$\theta_3 = \frac{ND}{ID} = \frac{MN}{IM} = \frac{MN}{v}$$

From figure  $i$  is exterior angle  $i = \theta_1 + \theta_2$  and

$\theta_2$  is exterior angle thus  $\theta_2 = r + \theta_3$  or  $r = \theta_2 - \theta_3$

Applying Snell's law at point N we get

$$n_1 \sin i = n_2 \sin r$$

for small angles of  $i$  and  $r$

$$n_1 i = n_2 r$$

$$n_1 (\theta_1 + \theta_2) = n_2 (\theta_2 - \theta_3)$$

$$n_1 \left( \frac{MN}{-u} + \frac{MN}{R} \right) = n_2 \left( \frac{MN}{R} - \frac{MN}{v} \right)$$

$$n_1 \left( \frac{1}{-u} + \frac{1}{R} \right) = n_2 \left( \frac{1}{R} - \frac{1}{v} \right)$$

By rearranging the terms

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

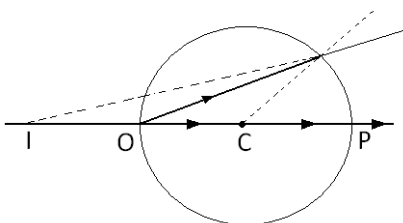
Above equation gives relation between object distance and image distance in terms of refractive index of the medium and radius of curvature of the spherical surface. It holds good for any spherical surface.

**curved surface magnification**

$$m = \frac{n_1 v}{n_2 u}$$

## Solved Numerical

Q) If a mark of size 0.2 cm on the surface of a glass sphere of diameter 10cm and  $n = 1.5$  is viewed through the diametrically opposite point, where will the image be seen and of what size?



Solution:

As the mark is on one surface, refraction will occur on other surface ( which is curved).

Further refraction is taking place from glass to air so,

$n_2 = 1$  ( air )  $n_1 = 1.5$  ( glass) object distance = diameter of sphere = -10cm and  $R = -5$  cm

Using formula

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{-10} = \frac{1 - 1.5}{-5}$$

$V = -20 \text{ cm}$

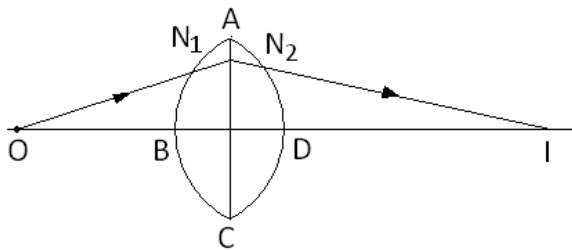
Hence image is at a distance of 20 cm from P towards Q

In case of refraction at curved surface magnification

$$m = \frac{n_1 v}{n_2 u} = \frac{1}{1.5} \times \frac{-20}{-10} = +3$$

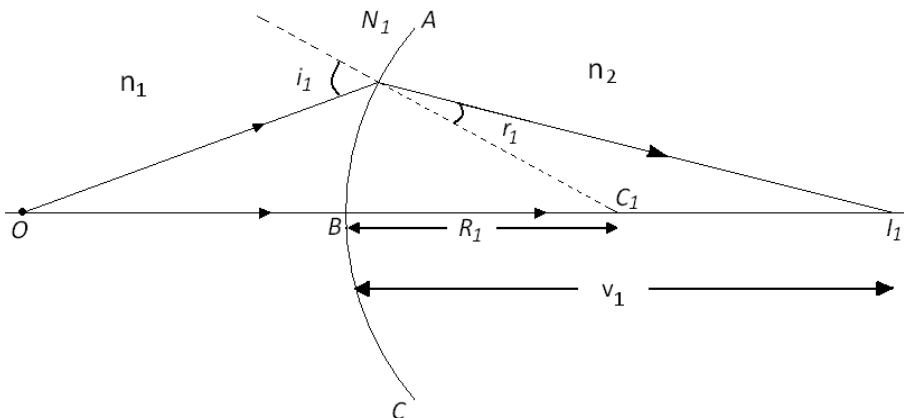
So, the image is virtual erect and of size  $h' = m \times h = 3 \times 0.2 = 0.6 \text{ cm}$

### Refraction by lens



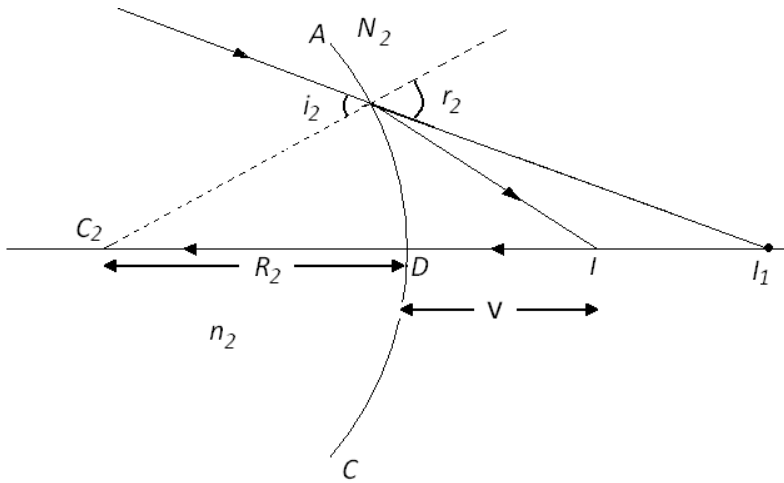
Lens shown in diagram have two curved surfaces. Surface ABC ( $N_1$ ) have radius of curvature  $R_1$ . Another surface ADC ( $N_2$ ) have radius of curvature  $R_2$ . Image formed by Surface  $N_1$  acts as an object for surface  $N_2$  and final image is formed at I

Consider surface ABC as shown in figure here object O is in medium of refractive index  $n_1$  and form image at  $I_1$  at a distance  $v_1$  in medium of refractive index  $n_2$ . from formula for refraction at curved surface.



$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \dots eq(1)$$

Now image  $I_1$  which is in medium of refractive index  $n_2$  acts like a object for surface ADC ( $N_2$ ) and forms image in medium of refractive  $n_1$  at I



here  $u = v_1$  and radius of curvature =  $R_2$

From formula for refraction at curved surface

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2} \quad \dots eq(2)$$

On adding equation (1) and (2) we get

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots eq(3)$$

$$\frac{1}{v} - \frac{1}{u} = \left( \frac{n_2}{n_1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots eq(4)$$

Here  $\frac{n_2}{n_1}$  is refractive index of lens material with respect to surrounding.

Lens-Makers's formula

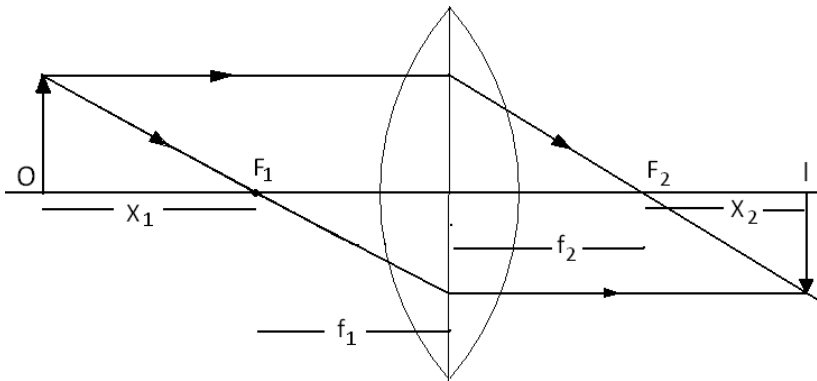
If  $u = \infty$  the image will be formed at  $f$  thus from equation 4

$$\frac{1}{f} = \left( \frac{n_2}{n_1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots eq(5)$$

Magnification

$$m = v/u$$

**Newton's formula:**



$x_1$  and  $x_2$  are known as extra focal distance and extra focal image distance.

$$x_1 \cdot x_2 = f_1 f_2$$

if  $f_1 = f_2 = f$  then

$$x_1 \cdot x_2 = f^2$$

**Power of lens:** It is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre

$$\tan \delta = \frac{h}{f}$$

If  $h = 1$  then

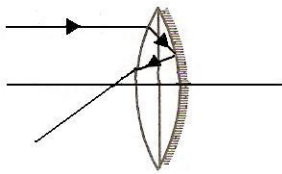
$$\tan \delta = \frac{1}{f}$$

For small value of  $\delta$

$$\delta = \frac{1}{f}$$

Thus  $P = 1 / f$  SI unit for power of lens is diopter (D)  $1D = 1 \text{ m}^{-1}$

### Lens with one surface silvered



When one surface is silvered, the rays are reflected back at this silvered surface and set up acts as a spherical mirror

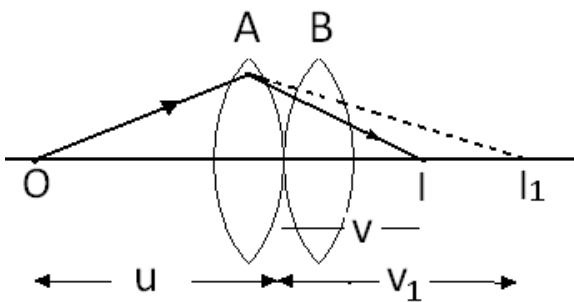
Focal length is given by formula

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$$

Here  $f_l$  is focal length of lens,  $f_m$  is focal length of mirror

In the above formula, **the focal length of converging lens or mirror is taken positive and that of diverging lens or mirror is taken as negative.**

### Combination of lenses in contact



Consider two lenses A and B of focal length  $f_1$  and  $f_2$  placed in contact with each other. Let the object be placed at a point O beyond the focus of the first lens A.

The first lens produces the image at  $I_1$ . Since the image is real. It serves as object for second lens B produces image at I

The direction of rays emerging from the first lens gets modified in accordance with the angle at which they strike the second lens.

We assume the optical centres of the thin lens coincident.

For image formed by lens A. we get

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

For the image formed by second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

Adding above equations

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If two lens-system is regarded as equivalent to a single lens of focal length  $f$ , we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

So we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The derivation is valid for any number of thin lenses in contact

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \dots \dots \dots$$

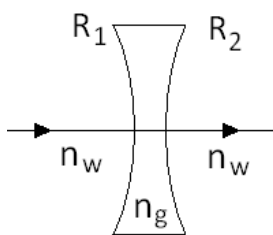
In terms of equivalent power

$$P = P_1 + P_2 + P_3 + \dots \dots \dots$$

Image form by first lens is object for second lens it implies that total magnification for combination of lenses is

$$m = m_1 m_2 m_3 \dots \dots \dots \text{ (here } m_1 \text{ and } m_2 \dots \text{etc are magnification of individual lenses)}$$

### Solved Numerical



Q) Calculate the focal length of a concave lens in water (Refractive index =  $4/3$ ) if the surface have radii equal to 40cm and 30 cm (refractive index of glass = 1.5)

Solution:  $R_1 = -30\text{cm}$   $R_2 = +40\text{cm}$

We have

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{4/3} - 1\right) \left(\frac{1}{-30} - \frac{1}{40}\right)$$

$$\frac{1}{f} = -\frac{960}{7} = -131.7 \text{ cm}$$

Q) A plano-covex lens has focal length 12cm and is made up of glass with refractive index 1.5. Find the radius of curvature of its curved side

Solution: Let  $R_1$  be the radius of curvature,  $R_2 = \infty$ ,  $f = +12\text{cm}$

From formula for focal length



$$\frac{1}{f} = (n_2 - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{12} = \left( \frac{1.5}{1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{\infty} \right)$$

$$R_1 = 6 \text{ cm}$$

Q) A magnifying lens has a focal length of 10cm (i) Where should the object be placed if the image is to be 30cm from the lens? (ii) What will be the magnification

Solution: For a convergent lens, If image formed on object side,

Thus  $v = -30 \text{ cm}$   $f = +10 \text{ cm}$

Let  $x$  be the distance of object

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{-30} - \frac{1}{-x}$$

$$x = 7.5 \text{ cm}$$

(ii) magnification  $m = v/u = -30/-7.5 = +4$

Thus image is virtual erect and four times the size of the object.

Q) An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of the image formed is 50 cm, find the distance between the object and the image

Solution

We have

$$|m| = h'/h = v/u = 50/25 = 2$$

There are two possibilities

(i) if the image is inverted (i.e.) real

$$M = v/u = -2 \text{ or } v = -2u$$

Let  $x$  be the object distance, in this case

We have  $u = -x$ ,  $v = +2x$   $f = +30 \text{ cm}$

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{2x} - \frac{1}{-x}$$

$$x = 45 \text{ cm then } v = 90 \text{ cm}$$

Hence distance between object and image is  $= 45 + 90 = 135$

(ii) If the image is erect (i.e. virtual)

$$m = v/u = +2$$

Let  $x'$  be the object distance, we have

$$u = -x'; v = -2x'; f = 30 \text{ cm}$$

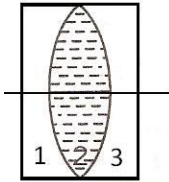
Using the lens formula, we have

$$\frac{1}{30} = \frac{1}{-2x'} - \frac{1}{-x'}$$

$$x' = 15 \text{ cm}$$

Hence distance between object and image = 15 cm

Q) Two plano-concave lenses made of glass of refractive index 1.5 have radii of curvature 20 cm and 30cm. they are placed in contact with curved surface towards each other and the space between them is filled with liquid of refractive index 4/3. Find the focal length of the system



Solution: As shown in figure, the system is equivalent to combination of three lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

By lens maker's formula

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{\infty} - \frac{1}{20}\right) = \frac{1}{40} \text{ cm}$$

$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right) \left(\frac{1}{20} - \frac{1}{-30}\right) = \frac{5}{180} \text{ cm}$$

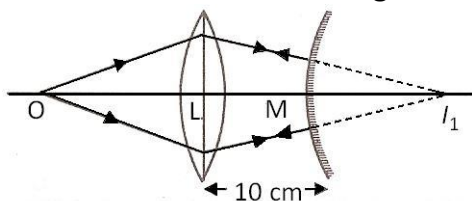
$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{-30} - \frac{1}{\infty}\right) = \frac{-1}{60} \text{ cm}$$

Now

$$\frac{1}{F} = \frac{1}{40} + \frac{1}{180} + \frac{-1}{60} = \frac{-1}{72}$$

$$F = -72 \text{ cm}$$

Q) A point object is placed at a distance of 12 cm on the axis of convex lens of focal length 10cm. On the other side of the lens, a convex mirror is placed at the distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of the mirror.?



Solution :

For the refraction at the convex lens, we have  $u = -12$  cm,  $v = ?$   $f = +10$  cm

Using lens formula, we have

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-12}$$

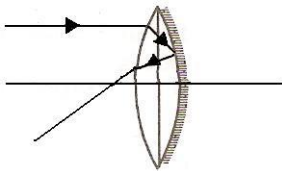
$V = +60$ cm

Thus, in the absence of convex mirror, convex lens will form the image at  $I_1$ , at a distance 60 cm behind the mirror. This image will act as object for mirror t, object distance for mirror =  $60 - 10 = +50$ cm

Now as the final image I is formed at the object itself, indicates that the rays on the mirror are incident normally i.e image  $I_1$  is at the centre of curvature of mirror

So  $R = 50$  cm and  $f = R/2 = 50/2 = +25$ cm

Q) One face of an equiconvex lens of focal length 60cm made of glass ( Refractive index 1.5) is silvered. Does it behave like a concave mirror or convex mirror



Solution : Let  $x$  be the radius of curvature of each surface from lens maker's formula

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{60} = (1.5 - 1) \left(\frac{1}{x} - \frac{1}{-x}\right)$$

$$\frac{1}{60} = 0.5 \times \frac{2}{x}$$

$$X = 60 \text{ cm}$$

Let  $f$  be the focal length of the equivalent spherical mirror. Then

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$$

Since lens is equiconvex

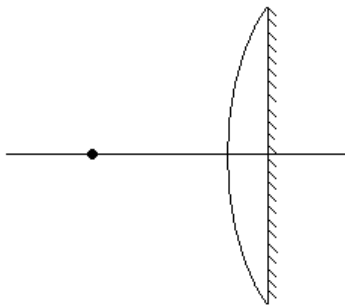
$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m}$$

Here  $f_l = +60$  cm (convex lens) ,  $f_m = R/2 = +30$  (concave mirror)

$$\frac{1}{f} = \frac{2}{60} + \frac{1}{30} = \frac{1}{15}$$

$F = +15$ cm cm

The positive sign indicates that the resulting mirror is concave



Q) The plane surface of a plano-convex lens of focal length 60 cm is silvered . A point object is placed at a distance 20cm from the lens. Find the position and nature of the final image formed

Solution:

Let  $f$  be the focal length of the equivalent spherical mirror

We have

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$$

Or

$$\frac{1}{f} = \frac{2}{f_l} + \frac{1}{f_m}$$

Here  $f_l = +60$  cm,  $f_m = \infty$

$$\frac{1}{f} = \frac{2}{60} + \frac{1}{\infty} = \frac{1}{30}$$

Or  $f = +30$  cm

The problem is reduced to a simple case where a point object is placed in front of a concave mirror/

Now, using the mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

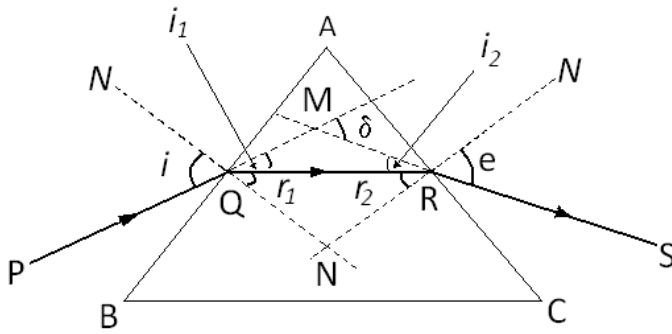
$$\frac{1}{-30} = \frac{1}{v} + \frac{1}{-20}$$

$V = 60$  cm

## SECTION III

# REFRACTION THROUGH A PRISM

### REFRACTION THROUGH A PRISM



PQ is incident ray and angle made by PQ with normal is  $i$ . QR is refracted ray at surface BA and angle of refraction is  $r_1$

For surface AC ray QR is incident ray and angle of incidence is  $r_2$ . RS refracted ray and angle is called angle of emergence

The angle between the emergent ray

and RS and the direction of incident ray PQ is called the angle of deviation  $\delta$ .

In quadrilateral AQNR, two angles ( at vertices Q and R ) are right angles. Therefore the sum of other angles of quadrilateral is  $180^\circ$

$$\angle A + \angle QNR = 180^\circ \text{ --eq(1)}$$

From the triangle QNR

$$r_1 + r_2 + \angle QNR = 180^\circ \text{ --eq(2)}$$

from equation (1) and (2)

$$r_1 + r_2 = A \text{ --eq(3)}$$

Now  $\delta = i_1 + i_2$  ( as  $\delta$  is exterior angle ) --eq(4)

Now  $i_1 = (i - r_1)$  and  $i_2 = (e - r_1)$  ( from geometry of figure)

From equation 3

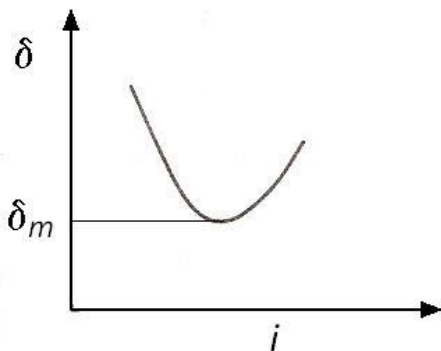
$$\delta = (i - r_1) + (e - r_1)$$

$$\delta = (i + e) - (r_1 + r_1)$$

$$\delta = (i + e) - A \text{ --eq(5)}$$

Thus , the angle of incidence depends on the angle of incidence.

From the graph  $\delta$  vs  $i$  we can find that for every angle of deviation there are two values of angle of incidence. It means that vale of  $i$  and  $e$  are interchangeable. Except  $i=e$  . At  $i=e$  the angle of deviation is minimum denoted by  $\delta_m$  or  $D_m$ , at angle of minimum deviation ray inside the prism becomes parallel.



### Calculation of refractive index of Prism

By applying Snells law at the surface BA we get

$$n_1 \sin i = n_2 \sin r_1$$

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin i}{\sin r}$$

At angle of minimum deviation  $i = e$  implies  $r_1 = r_2 = r$  From equation (3)

$$2r = A \text{ or } r = A/2 \text{ ---eq(6)}$$

From equation (5)

$$D_m = 2i - A \text{ or } i = (A + D_m)/2 \text{ ---eq(7)}$$

Substituting values from equation 6 and 7

$$n_{21} = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$$

For small angle prism  $D_m$  is also very small and we get

$$n_{21} = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}} = \frac{\frac{A + D_m}{2}}{\frac{A}{2}} = \frac{A + D_m}{A}$$

$$D_m = (n_{21} - 1) A$$

It implies that, thin prism do not deviate light much

### Solved Numerical

Q) A ray of light falls on one side of a prism whose refracting angle is  $60^\circ$ . Find the angle of incidence in order that the emergent ray just graze the other side

( Refractive index =  $3/2$ )

Solution: Given  $A = 60^\circ$ ,  $e = 90^\circ$

$\therefore r_2 = C$  the critical angle of the prism

Now  $n = 1/\sin C$

Or  $\sin C = 1/n = 2/3$

$C = 41^\circ 49'$

Again,  $A = r_1 + r_2$

$r_1 = A - r_2 = 60^\circ - 41^\circ 49' = 18^\circ 11'$

For the refraction at surface AB we have

$$n = \frac{\sin i}{\sin r_1}$$

$$\sin i = n \sin r_1$$

$$\sin i = 1.5 \times \sin 18^\circ 11'$$

$$\sin i = 1.5 \times 0.312 = 0.468$$

$$i = 27^\circ 55'$$

Q) The refractive index of the material of a prism of refracting angle  $45^\circ$  is 1.6 for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as they come out of the prism

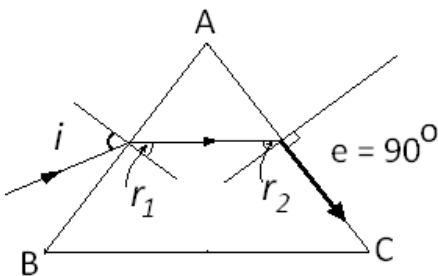
Solution:

Given  $A = 45^\circ$ ,  $n = 1.6$

We have

$$\sin C = 1/n$$

$$\sin C = 1/1.6$$



$$C = 38.68^\circ$$

For total internal reflection not take place at the surface AC, we have

$$r_2 \leq C$$

$$\text{or } (r_2)_{\max} = C$$

$$\text{Now } r_1 + r_2 = A$$

$$\text{Or } r_1 = (A - r_2)$$

$$\text{Or } (r_1)_{\min} = A - (r_2)_{\max} = 45^\circ - 38.68^\circ = 6.32^\circ$$

For the refraction at the first face

We have

$$n = \frac{\sin i_1}{\sin r_1}$$

$$\text{Or } \sin i_1 = n \sin r_1$$

$$\sin i_1 = 1.6 \times \sin (6.32^\circ) = 0.176$$

$$i_1 = 10.14^\circ$$

Q) Find the minimum and maximum angle of deviation for a prism with angle  $A = 60^\circ$  and  $n = 1.5$

Solution : Minimum deviation:

The angle of minimum deviation occurs when  $i = e$  and  $r_1 = r_2$  and is given by

$$n = \frac{\sin \frac{(A + \delta_m)}{2}}{\sin \frac{A}{2}}$$

$$\delta_m = 2 \sin^{-1} \left( n \sin \frac{A}{2} \right) - A$$

Substituting  $n = 1.5$  and  $A = 60^\circ$ , we get

$$\delta_m = 2 \sin^{-1} (0.75) - 60 = 37^\circ$$

Maximum deviation

The deviation is maximum when  $i = 90^\circ$  or  $e = 90^\circ$

Let  $i = 90^\circ$

$$r_1 = C = \sin^{-1} (1/n)$$

$$r_1 = \sin^{-1} (2/3) = 42^\circ$$

$$r_2 = A - r_1 = 60^\circ - 42^\circ = 18^\circ$$

Using

$$\frac{\sin r_2}{\sin e} = \frac{1}{n}$$

$$\sin e = n \sin r_2$$

$$e = 28^\circ$$

$$\therefore \text{Deviation} = \delta_{\max} = (i + e) - A = 90 + 28 - 60 = 58^\circ$$

## Dispersion by a prism

The phenomenon of splitting of light into its component colours is known as dispersion.

Different colours have different wavelengths and different wavelengths have different velocity in transparent medium.

Longer wave length have more velocity compared to shorter wavelength. Thus refractive index is more for shorter vale length than longer wavelength.

Red colour light have highest wave length in visible light spectrum and hence lowest refractive index.

Violet colour light have smallest wave length in visible light spectrum., it have maximum refractive index.

Thick lens could be considered as maid of many prism, therefore, thick lens shows chromatic aberration due to dispersion of light

## Some natural phenomena due to sunlight

### THE RAINBOW

#### Primary rainbow

Rainbow is formed due to two times refraction and one time total internal reflection in water drop present in atmosphere

Sunlight is first refracted as it enters a raindrop, which causes the different wavelengths of white light to separate. Longer wavelength of light ( red) are bent the least while the shorter wavelength ( violet) are bent the most. Next, these component rays strike the inner surface of the water droplet and gets internally reflected if the angle of incidence is greater than the critical angle. The reflected light is again refracted as it comes out of the drop.

It is found that violet light emerges at an angle  $40^\circ$  related to the incoming sunlight and red light emerges at an angle of  $42^\circ$  . For other colours , angle lies between these two values.

#### Secondary rainbow

When light rays undergoes two internal refraction inside the rain drop, a secondary raindrop is formed. Its intensity is less compared to primary due to four stem process. Violet ray makes an angle of  $53^\circ$  and red light rays makes  $50^\circ$  with related to incoming sunlight. The order of colours is reversed

#### Rayleigh scattering

The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as **Rayleigh scattering**

#### Why sky is blue

As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as **Rayleigh scattering**. Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, having a shorter wavelength. But since our eyes are more sensitive to blue than violet, we see the sky blue.



**Why clouds look white**

Large particles like dust and water droplets present in the atmosphere behave differently. The relevant quantity here is the relative size of the wavelength of light  $\lambda$ , and do not follow **Rayleigh scattering**. For large scattering objects (for example, raindrops, large dust or ice particles) all wavelengths are scattered nearly equally. Thus, clouds which have droplets of water with larger particle are generally white.

**At sunset or sunrise, Sun looks reddish**

Sun rays have to pass through a larger distance in the atmosphere (Fig. 9.28). Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.

## SECTION III

### OPTICAL INSTRUMENTS

## OPTICAL INSTRUMENTS

**Eye**

**Function of ciliary muscles:** The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles

For example, when the muscle is relaxed, the focal length is about 2.5 cm and objects at infinity are in sharp focus on the retina. When the object is brought closer to the eye, in order to maintain the same image-lens distance ( $\cong 2.5$  cm), the focal length of the eye lens becomes shorter by the action of the ciliary muscles.

**Accommodation:** Property of eye to change the focal length as required is called accommodation.

**Retina:** The retina is a film of nerve fibers covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information.. For example, when the muscle is relaxed, the focal length is about 2.5 cm and objects at infinity are in sharp focus on the retina. When the object is brought closer to the eye, in order to maintain the same image-lens distance ( $\cong 2.5$  cm), the focal length of the eye lens becomes shorter by the action of the ciliary muscles.

**Least distance of distinct vision:** If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred. The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision, or the near point. The standard value for normal vision is taken as 25 cm. (Often the near point is given the symbol D.) This distance increases with age, because of the decreasing effectiveness of the ciliary muscle and the loss of flexibility of the lens. The near point may

be as close as about 7 to 8 cm in a child ten years of age, and may increase to as much as 200 cm at 60 years of age.

**Presbyopia:** If an elderly person tries to read a book at about 25 cm from the eye, the image appears blurred. This condition (defect of the eye) is called presbyopia. It is corrected by using a converging lens for reading.

**Myopia:** the light from a distant object arriving at the eye-lens may get converged at a point in front of the retina. This type of defect is called nearsightedness or myopia. This means that the eye is producing too much convergence in the incident beam. To compensate this, we interpose a concave lens between the eye and the object, with the diverging effect desired to get the image focused on the retina

### Simple Microscope

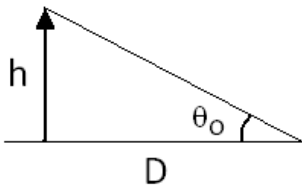
A simple magnifier or microscope is a converging lens of small focal length.

The least distance at which a small object can be seen clearly with comfort is known as near point (D) or distance of most distinct vision. For normal eye this distance is 25 cm. Suppose a linear object with height  $h_o$  is kept at near point ( i.e.  $u = D = 25$  cm) from eye. Let it subtend an angle  $\theta_o$  with eye .

**The magnification when the image is at infinity.**

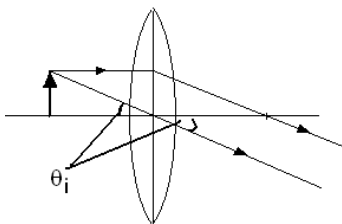
Now, if object is kept at the focal length (f) of a convex lens such that its virtual image is formed at a infinity. In this case we will have to obtain the *angular* magnification.

Suppose the object has a height  $h$ . The maximum angle it can subtend, and be clearly visible (without a lens), is when it is at the near point, i.e., a distance  $D$ . The angle subtended is then given by



$$\tan\theta_o \approx \theta_o = \frac{h}{D}$$

We now find the angle subtended at the eye by the image when the object is at  $u$ . From the relations  $m = \frac{h'}{h} = \frac{v}{u}$



$$h' = \frac{v}{u}h$$

we have the angle subtended by the image

$$\tan\theta_i \approx \theta_i = \frac{h'}{-v} = \frac{v}{u}h \left( \frac{1}{-v} \right) = \frac{h}{-u}$$

The angle subtended by the object, when it is at  $u = -f$ .

$$\theta_o = \frac{h}{f}$$

The angular magnification is, therefore

$$m = \frac{\theta_i}{\theta_o} = \frac{D}{f}$$

**The magnification when the image is at the closest comfortable distance**

If the object is at a distance slightly less than the focal length of the lens, the image is virtual and closer than infinity.

Although the closest comfortable distance for viewing the image is when it is at the near point (distance  $D \cong 25$  cm), it causes some strain on the eye. Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye. The linear magnification  $m$ , for the image formed at the near point  $D$ , by a simple microscope can be obtained by using the relation.

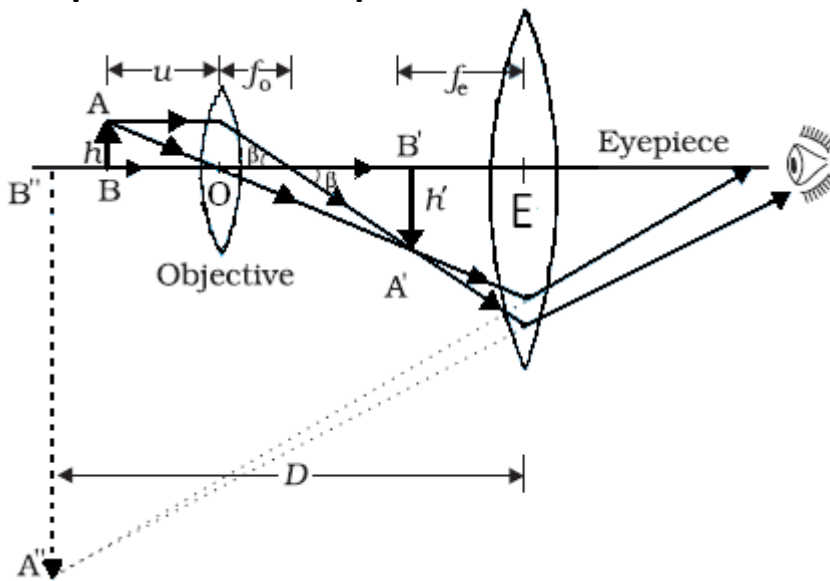
$$m = \frac{v}{u} = v \left( \frac{1}{v} - \frac{1}{f} \right) = \left( 1 - \frac{v}{f} \right)$$

Now according to our sign convention,  $v$  is negative, and is equal in magnitude to  $D$ . Thus, the magnification is

$$m = \left( 1 + \frac{D}{f} \right)$$

A simple microscope has a limited maximum magnification ( $\leq 9$ ) for realistic focal lengths

**Compound microscope**



A simple microscope has a limited maximum magnification ( $\leq 9$ ) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a *compound microscope*.

A schematic diagram of a compound microscope is shown in Fig..The lens nearest the object, called the *objective*, And lens near to eye is called *eye piece*

Objective forms a real, inverted, magnified image of the object.  $A'B'$ .

This serves as the object for the *eyepiece*, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual  $A''B''$  the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length  $f_e$ ) is called the **tube length** ( $L$ ) of the compound microscope

**magnification**

The ray diagram of Fig. shows that the (linear) magnification due to the objective,

$$\tan\beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)$$

$$m_o = \frac{h'}{h} = \frac{L}{f_o}$$

Here  $h'$  is the size of the first image, the object size being  $h$  and  $f_o$  being the focal length of the objective.

The first image is formed near the focal point of the eyepiece. Magnification due to eye piece which behaves as simple microscope is given by

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

When the final image is formed at infinity, the angular magnification due to the eyepiece

$$m_e = \left(\frac{D}{f_e}\right)$$

Thus, the total magnification, when the image is formed at infinity, is

$$m = m_o m_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right)$$

Thus, the total magnification, when the image is formed at near point, is

$$m = m_o m_e = \left(\frac{L}{f_o}\right) \left(1 + \frac{D}{f_e}\right)$$

**Telescope**

The telescope is used to provide angular magnification of distant objects.

Lens towards object is objective have larger focal length than eye piece.

Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.

**The magnifying power  $m$  is the ratio of the angle  $\beta$  subtended at the eye by the final image to the angle  $\alpha$  which the object subtends at the lens or the eye.** Hence

$$m \approx \frac{\beta}{\alpha} = \frac{h f_o}{f_e h} = \frac{f_o}{f_e}$$

In this case, the length of the telescope tube is  $f_o + f_e$ .

Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations.

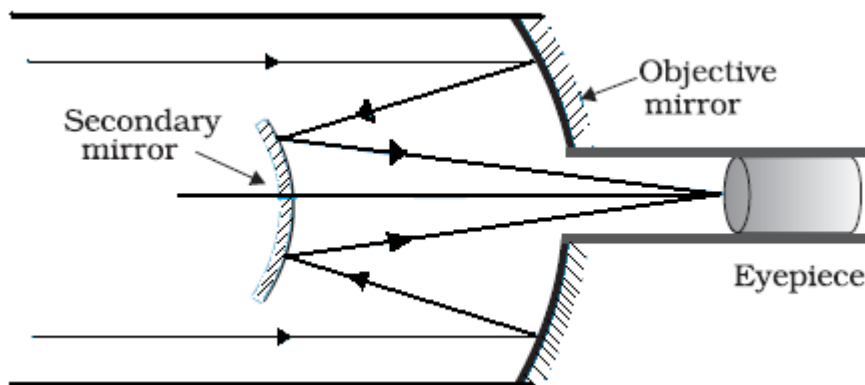
**If the final image is formed at the least distance of distinct vision, then magnifying power is**

$$m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed.

The **resolving power**, or the ability to observe two objects distinctly, which are in very nearly the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes

## Reflecting telescope



Telescopes with mirror objectives are called *reflecting* telescopes.

They have several advantages over refracting telescope

First, there is no chromatic aberration in a mirror.

Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed.

Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim.

One obvious problem with a reflecting telescope is that the objective mirror focuses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage). This is what is done in the very large 200 inch (~5.08 m) diameters, Mt. Palomar telescope, California.

The viewer sits near the focal point of the mirror, in a small cage. Another solution to the problem is to deflect the light being focused by another mirror. One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, is shown

## Solved Numerical

Q) A compound microscope with an objective of 2.0 cm focal length and an eye piece of 4.0 cm focal length, has a tube length of 40cm. Calculate the magnifying power of the microscope, if the final image is formed at the near point of the eye

Solution Formula for magnification when image is formed at near point of eye

$$m = m_o m_e = \left(\frac{L}{f_o}\right) \left(1 + \frac{D}{f_e}\right)$$

$$m = \left(\frac{40}{2}\right) \left(1 + \frac{25}{4}\right) = 145$$

Q) A compound microscope consists of an objective of focal length 1cm and eyepiece of focal length 5cm separated by 12.2 cm (a) At what distance from the objective should an object should be placed to focus it properly so that the final image is formed at the least distance of clear vision ( 25cm)? (b) Calculate the angular magnification in this case.

Solution : From lens formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Here  $f = 1 \text{ cm}$  ,;  $u = u_e$  ,  $v = - 25\text{cm}$

$$\frac{1}{5} = \frac{1}{25} + \frac{1}{u_e}$$

$U_e = -4.2 \text{ cm}$

$V_o = L - |u_e| = (12.2 - 4.2) \text{ cm} = 8 \text{ cm}$

From lens formula

$$\frac{1}{1} = \frac{1}{8} + \frac{1}{u_o}$$

$U_o = -1.1 \text{ cm}$

From formula for angular magnification =

$$m = \left(\frac{v_o}{u_o}\right) \left(1 + \frac{D}{f_e}\right)$$

$$m = \left(\frac{8}{-1.1}\right) \left(1 + \frac{25}{5}\right) = -43.6$$

Q) The separation between the objective (  $f = 0.5\text{cm}$ ) and the eyepiece (  $f = 5\text{cm}$ ) of a compound microscope is 7cm. Where should a small object be placed so that the eye is least strained to see the image? Find the angular magnification produced by the microscope

Solution: The eye will strain least if the final image is formed at infinity> In this case the image formed by the objective shall fall at the focus of the eye piece.

Now  $v_o = L - v_e = ( 7 - 5) \text{ cm} = 2\text{cm}$

From lens formula for objective lens

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{0.5} = \frac{1}{2} + \frac{1}{u_o}$$

$$u_o = -2/3 \text{ cm}$$

angular magnification for image at infinity

$$m = \left(\frac{v_o}{u_o}\right) \left(\frac{D}{f_e}\right)$$

$$m = \left(\frac{2 \times 3}{-2}\right) \left(\frac{25}{5}\right) = -15$$

Q) An astronomical telescope, in normal adjustment position, has magnifying power 5. The distance between the objective and the eyepiece is 120cm. Calculate the focal lengths of the objective and of the eyepiece.

Solution:  $m = f_o / f_e$

$$f_o = m \times f_e$$

$$f_o = 5 f_e$$

$$f_o + f_e = 120$$

thus  $f_e = 20 \text{ cm}$  and  $f_o = 100 \text{ cm}$

Q) The focal length of the objective of an astronomical telescope is 75 cm and that of the eye piece is 5 cm. If the final image is formed at the least distance of distinct vision from eye, calculate the magnifying power of telescope

Solution:

$$m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

$$m = \frac{75}{5} \left(1 + \frac{5}{25}\right) = 18$$

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