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LESSON 1

ELECTRIC CHARGES AND FIELDS

SECTION I

ELECTRIC CHARGE

Electric charge and its characteristic

(i) ***Electric charges is the fundamental intrinsic property due to which electric force acts***

(ii) Electric charges are two types. Traditionally charge on proton is positive and charge on electron negative. Magnitude of charge on both is same $1.6X10^{-19}$C

(iii) Fundamental charge is the charge on the electron or proton denoted by e.

(iv) Quantization of charges: ***Magnitude of all charges are found to be integral multiple of fundamental charge thus If Q is total charge then Q = ne here n = 1, 2, 3 ,…***

(v) Law of conservation of electric charge state that "The ***algebraic sum of electric charge in an electrically isolated system always remains constant irrespective of any process taking place.*** Or in other words "***In an electrically isolated system only those processes are possible in which charges of equal magnitude and opposite types are either produced or destroyed.*** Example Before rubbing glass rod on silk algebraic sum of charge is zero. On rubbing and separating the rod from silk, we find equal and opposite amount of charge is developed on silk and glass rod.

(vi) Two charges exerts equal and opposite force on each other. Like charges repeals while unlike attracts

(vii) S.I Unit of charge is "coulomb' denoted by C. CGS unit of charge esu
1 coulomb = $3X10^9$esu

(viii) Charge cannot exists without mass though mass can exist without charge

(ix) Charge is invariant: This means that ***charge is independent of frame of reference. i.e. change of the body does not charge with whatever be its speed***

Ways of charging body

(A) Charging by friction:

When two bodies are rubbed together, a transfer of electrons take place from one body to another. The body from which electrons have been transferred is left with an excesses of positive charge, so get positively charged. The body which receives the electrons becomes negatively charged.

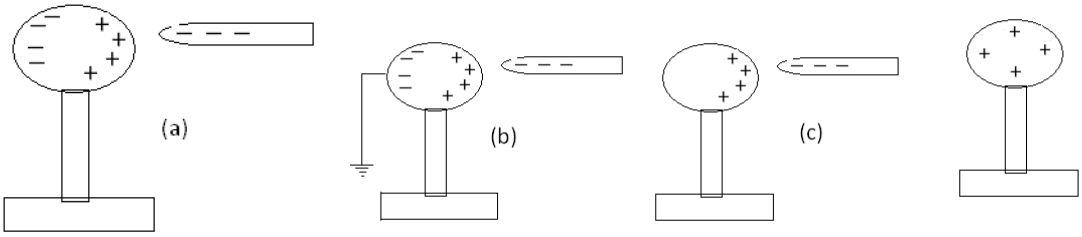
"The positive charge and negative charges produced by rubbing are always equal in magnitude"

When glass rod is rubbed on silk, glass rod loses its electrons and gets positive charges, while silk acquires equal negative charges.

An ebonite or plastic rod acquires a negative charge, if it is rubbed with wool. The piece of wool acquires an equal positive charge

(B) Charging by electrostatic induction

If a negatively charge rod is brought near the conductor mounted on insulated base as free electrons of conducting spheres close to rod experiences a force of repulsion and go to the other part of the sphere as shown in fig a .



Consequently the part of sphere close to rod becomes positively charge due to deficiency of electrons in that region.
As shown in figure b when the sphere is connected to the earth through a conducting wire, the some of the electrons of the spheres will flow to the ground.
As shown in figure c, even if the connection with the earth is removed, the sphere retains the positive charge. When the negatively charged rod is moved away from the sphere, the electrons get redistributed on the sphere such that the same positive charge is spread all over the surface of the sphere as shown in figure d
Important points regarding electrostatic induction
(a) Inducing body neither gains nor loses charges
(b) The nature of induced charge is always opposite to that of inducing charge
(c) Induced charge can be lesser or equal to inducing charge but it is never greater than the inducing charge
(d) Induction takes place only in bodies (either conducting or non conducting) and not particles

(C) Charging by conduction

Let us consider two conductor, one charged and other uncharged. We bring the conductors in contact with each other. The charge under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. If charged and uncharged conductors are of same size charge will be equally divided if separated after contact.

Solved numerical

Q) A copper penny has a mass of 3.1g. Being electrically neutral, it contains equal amount of positive and negative charges. What is the magnitude of these? A copper atom has a positive nuclear charge of $4.6X10^{-18}$ C. Atomic weight of copper is 64 g/mole and Avogadro's number is $6X10^{23}$ atoms/mole
Solution : 1 mole of copper i.e 64 g of copper has $6X10^{23}$ atoms. Therefore, the number of atoms in copper penny of 3.1 g is

$$\frac{6\times10^{23}}{64}\times 3.1 = 2.9X10^{22}$$

One atom of copper has each positive and negative charge of $4.6X10^{-18}$C. So each charge on the penny is $(4.6X10^{18})$ X $(2.9X10^{22}) = 1.3X10^{5}$ C

QUESTIONS (A)

Q) How many number of protons of the charge is equivalent to a 1μC?
Q) To identical metal spheres of equal radius are taken one is charged with 100 electrons and other is with 60 protons. When the two spheres are brought in contact and the separated , What will be the charges on each spheres
Q) What will happen when a charged body is placed near an uncharged body.
Q) Can we charge a body having charge $10X10^{-19}$C
Q) How many electrons must be removed from a piece of metal so as to leave it with a positive charge of 10^{-7} coulomb?
Q) A metal sphere is suspended through a nylon thread. When another charged sphere (identical to A) is brought near A and kept at a distance d, a force of repulsion F acts between them. Now A is brought in contact with identical uncharged sphere D and then they are separated from each other. What will be the force between the sphere A and B when they are at a distance d/2 [Ans F]
Q) Write the conservation law of charges
Q) The electric charge of a macroscopic body is either a surplus or deficit of electrons. Why not protons.
Q) How does the force between two point charges, if the dielectric constant of the medium in which they are kept decreases?
Q) Can two bodies, both carrying same type of charge be attracted to each other? [hint yes, if second body have very large charge than to first, explain on the basis of induction]
Q) Write down the value of constant of proportionality $\frac{1}{4\pi\varepsilon_0}$ involved in expression for Coulomb's force between the two charges
Q) Write down the value of absolute permittivity of free space (vacuum), give its dimensional formula
Q) Name the experiment which established quantum nature of electric charge?
[Millikan's oil drop experiment for measurement of electric charge]
Q) Can a body have a charge of $0.8X10^{-19}$C
Q) Why does an ebonite rod get negative charge on rubbing with fur
Q) What does $q_1 + q_2 = 0$ signify in electrostatics?
Q) What is the basic cause of quantization of charge
Q) What is the least possible value of charge
Q) Does the motion of body affect its charge?
Q) What is the cause of charging?
Q) How mass of a body is affected on charging
Q) Making use of conservation of charges, identify the element x in the following nuclear reactions
(i) $H^1 + Be^9 \rightarrow X + n$
(ii) $C^{12} + H^1 \rightarrow X$
(iii) $N^{15} + H^1 \rightarrow He^4 + X$
Q) Ordinary rubber is an insulating. But the special rubber tyres of aircrafts are mage slightly conducting. Why this is necessary

Q) Vehicle carrying inflammable material usually have metallic ropes touching the ground during motion why?
Q) Give two point differences between charge and mass
Q) How you can charge a metal sphere positively without touching it

Coulomb's law :

" Two point charges repel or attract each other with force which is directly proportional to the product of the magnitude of their charges and inversely proportional to the square of the distance between them"

Let 'r' be the distance between two point charges q_1 and q_2 the according to Coulomb's law $F \propto \frac{|q_1||q_2|}{r^2}$ or $F = \frac{K|q_1||q_2|}{r^2}$,

K is proportionality constant . The value of K depends on the medium in which two point charges are placed

In SI system K = $\frac{1}{4\pi\varepsilon_0}$ for vacuum (or air)

The constant ε_0 (=8.85X10^{-12} Nm^{2}C^{-2}) is called "permittivity" of the free space

Value of K = 9X10^9

Permittivity of medium

If medium between the charges is not a vacuum (or air) then

$$\boxed{F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|q_1||q_2|}{r^2}}$$

Where ε_r is called ' relative permittivity" of medium , it is a dimension less quantity, it is also called as dielectric constant or specific inductive capacity

Term $\varepsilon = \varepsilon_0 \varepsilon_r$ are called as " absolute permittivity" or "permittivity" of the medium

Solved numerical

Q) The repulsive force between two particles of same mass and charge , separated by a certain distance equal to the weight of one of them. Find the distance between them .

Mass of particle = 1.6X10^{-27} kg, charge on particle = 1.6X10^{-19}C, g = 10 ms^{-2}

Solution: Here repulsive force between two particles = weight of one of particle

$$\frac{Kq_1q_2}{r^2} = mg$$

$$r = \left(\frac{Kq_1q_2}{mg}\right)^{1/2}$$

On substituting values we get r = 1.44X10^{-2}

Q) Two point charges placed at certain distance r in air exerts a force of F on each other. Then if dielectric of constant K is placed between the charges then what should be the thickness of the dielectric slab to get same force F between the charges

Solution

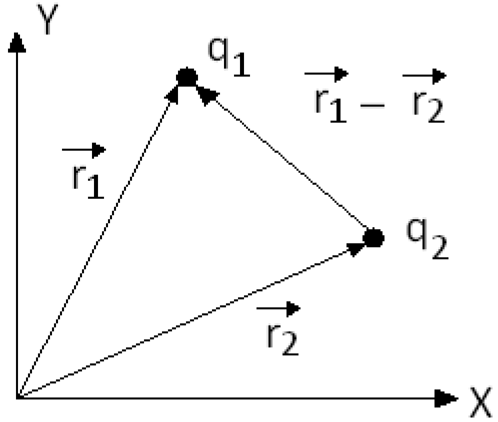
Let r’ be the thickness of dielectrics having dielectric constant k . since force remain same

$$\frac{Kq_1q_2}{r^2} = \frac{Kq_1q_2}{kr'^2}$$

Thus r’ = r/√k

Coulomb’s law in vector form

Force is a vector quantity, so Coulomb’s law can be represented in vector form as follow
Let two charges q_1 and q_2 are like charges (both positive or both negative charges)
Let $\mathbf{r_1}$ and $\mathbf{r_2}$ be the position vector s of the charge q_1 and q_2 .
Let $\mathbf{r_{12}}$ be the vector pointing q_2 to q_1, then displacement vector $\mathbf{r_{12}} = \mathbf{r_1 - r_2}$



According to coulomb’s law, force acting on charge q1 due to charge q2 is

$$\overrightarrow{F_{12}} = \frac{kq_1q_2}{(r_{12})^2}\hat{r}_{12}$$

Where $r_{12} = |\mathbf{r_1 - r_2}|$ is the distance between two charges and $\hat{r}_{12}$ us the unit vector of $\mathbf{r}_{12}$ in direction from q_2 to q_1

$$\hat{r}_{12} = \frac{\overrightarrow{r_1} - \overrightarrow{r_2}}{|\overrightarrow{r_1} - \overrightarrow{r_2}|}$$

∴

$$\overrightarrow{F_{12}} = \frac{kq_1q_2}{(|\overrightarrow{r_1} - \overrightarrow{r_2}|)^2}\frac{\overrightarrow{r_1} - \overrightarrow{r_2}}{|\overrightarrow{r_1} - \overrightarrow{r_2}|}$$

$$\overrightarrow{F_{12}} = \frac{kq_1q_2}{(|\overrightarrow{r_1} - \overrightarrow{r_2}|)^3}\overrightarrow{r_1} - \overrightarrow{r_2} \text{ .....(1)}$$

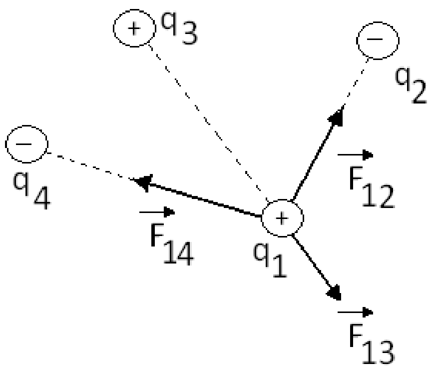
Above equation is valid for any sign of charge whether positive or negative
If charges are unlike then force will be negative indicating attractive force , if F is positive force is repulsive .
Force on q_2 due to q_1 in vector form can be represented as

$$\boxed{\overrightarrow{F_{12}} = \frac{kq_1q_2}{(|\overrightarrow{r_2} - \overrightarrow{r_1}|)^3}(\overrightarrow{r_2} - \overrightarrow{r_1})}$$

If we take out negative sign common from $\overrightarrow{r_1} - \overrightarrow{r_2}$ from equation (1), then we get $-(\overrightarrow{r_2} - \overrightarrow{r_1})$. Now $(\overrightarrow{r_2} - \overrightarrow{r_1})$ is a vector pointing from charge q_1 to q_2. Also $(|\overrightarrow{r_2} - \overrightarrow{r_1}|)^3$= $(|\overrightarrow{r_1} - \overrightarrow{r_2}|)^3$ thus from equation (1) and equation (2) we get $\overrightarrow{F_{21}}$ = - $\overrightarrow{F_{12}}$
Thus force of interaction between two bodies is equal and opposite Or Coulomb's law agree with Newton's Third Law

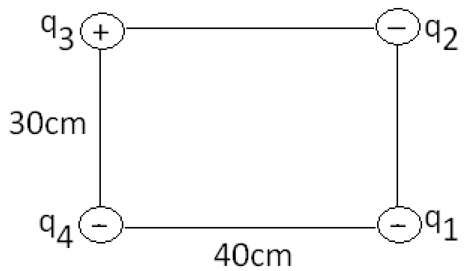
Principle of super position

According to the principle of super position, the force acting on one charge due to another is independent of the presence of other charges. So we can calculate the force separately for each pair of charges and then their vector sum or find the net force on any charge.



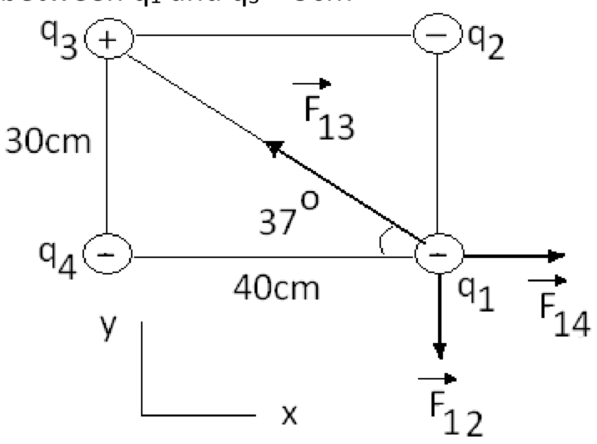
The figure shows a charge q_1 interacting with other charges. Thus, to find the force on q_1, we first calculate the forces exerted by each of the other charges, one at a time. The net force $\overrightarrow{F_1}$ on q_1 is simply the vector sum $\overrightarrow{F_1}$ = $\overrightarrow{F_{12}}$ + $\overrightarrow{F_{13}}$+$\overrightarrow{F_{14}}$+……….
Where $\overrightarrow{F_{12}}$ is the force on the charge q_1 due to the q_2 and so on

Solved numerical



Q) Find the net force on charge q_1 due to the three other charges as shown in figure. Take q1 = -5μC; q2 = -8 5μC; q3 =15 5μC; and q4 = -16

Solution : Angle between lines joining q_1, q_2 and q_1 ,q_3 is 37^O
The direction of the forces on q 1 and coordinate axes are as shown in figure and distance between q_1 and q_3 = 5cm



Now, F_{12}= $9 \times 10^9 \frac{q_1 q_2}{r^2}$

$F_{12} = \frac{9\times10^9\times(5\times10^{-6})(8\times10^{-6})}{(3\times10^{-1})^2} = 4N$

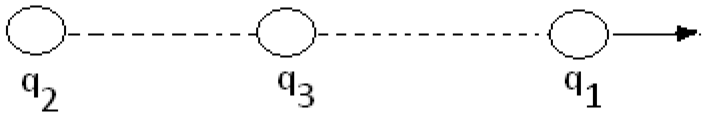
Similarly F_{13} = 2.7 N and F_{14} = 4.5 N
Forces in vector forms are
$\vec{F}_{12} = -4\hat{j}$, $\vec{F}_{13} = 2.7(-cos37\ \hat{\imath} + sin37\hat{j})$, $\vec{F}_{14} = 4.5\hat{\imath}$

Resultant force on q_1 is vector addition of all the forces

$$\vec{F}_1 = -4\hat{j} + 2.7(-cos37\,\hat{i} + sin37\hat{j}) + 4.5\hat{i}$$

$$\vec{F}_1 = 2.3\hat{i} - 2.38\hat{j}\text{ N}$$

Q) Three charges lie along the x axis as shown in figure. The positive charge q_1 = 15.0 μC is at c = 2.0 m and the positive charge q2 = 6.0 μC is at the origin. Where a negative charge q_3 must be placed on the x-axis such that the resultant force on it is zero?



Solution

Since q3 is negative and both q_1 and q_2 are positive, the force $\vec{F}_{31}$ and $\vec{F}_{32}$ are both attractive. Let x be the coordinate of q_3 we have

F31 = $\frac{kq_1q_3}{(2-x)^2}$ and F32 = $\frac{kq_3q_2}{(x)^2}$

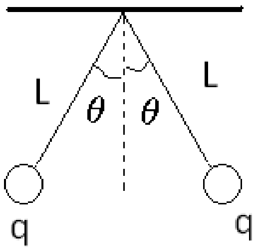
Since net force on the charge q_3 is a zero we have

$$\frac{kq_1q_3}{(2-x)^2} = \frac{kq_3q_2}{(x)^2}$$

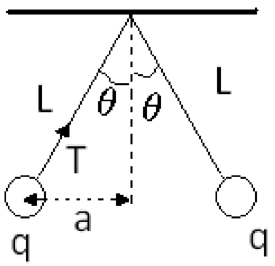
Or (4 – 4x +x) ($6X10^{-6}$) = x^2 ($15X10^{-6}$)

On solving quadric equation we get x = 0.775 m

Q) Two identical small charged spheres, each having a mass of 3.0X10 -2 kg, hang in equilibrium as shown below, if the length of each string is 0.15 m and the angle θ = 5°, find the magnitude of the charge on each sphere.

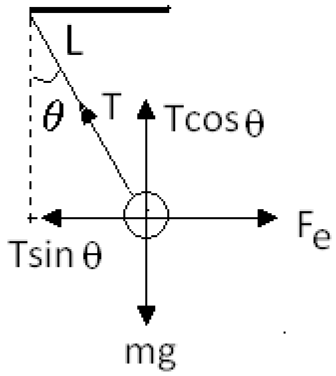


Solution :



From figure a = L sinθ = 0.15sin5 = 00.013m . Hence spheres are at separation 2a = 0.26m

From FBD of one sphere



Since the sphere is in equilibrium, the resultant forces in the horizontal and vertical direction must separately added up to zero, thus

Tsinθ = F_e = 0 –(i)

Tcosθ = mg --(ii)

Dividing equation (i) by (ii) we get

tanθ = F_e/mg or Fe = mg tanθ

F_e =(3X10-2) X(9.8)(tan5)

F_e = 2.6X10 $^{-2}$ N

Let q be charge on each sphere, According to coulomb's law

F_e= $\frac{9\times10^9 q^2}{r^2}$

2.6X10^{-2} = $\frac{9\times10^9 q^2}{(0.026)^2}$

Or q = 4.4X10^{-8}C

Force due to continuous charge distributions

To find the force exerted by a continuous charge distribution on a point charge, we divide the charge into infinitesimal charge element. Each infinitesimal charge element is then considered as a point charge. The magnitude of the force dF exerted by the charge dq on the charge q_0 is given by

$$dF = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|dq||q_0|}{r^2}$$

Where r is the distance between dq and q_0 . The total force is then found by adding all the infinitesimal force element, which involves integral

Each type of the charge distribution is described in table below by an appropriate Greek-letter parameter λ, σ, ρ

Charge distribution	Relative parameter	SI unit	Charge on element
Along a line	λ, charge per unit length Q/L Q is charge L is length	C/m	dq = λ dx
On surface	σ , charge per unit area Q/A Q is charge , A is area	C/m^2	dq = σ dA

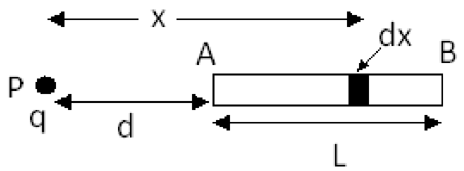
Throughout volume	ρ, charge per unit volume Q/V Q is charge V is volume	C/m^3	dq = ρ dV

Note : charge distribution is continuous but may not be uniform thus charge distribution is function of position

Solved numerical

Q) A point charge is situated at a distance d from one end of a thin non-conducting rod of length L having a charge Q (uniformly distributed along the length) as shown. Find the magnitude of the electric force between the two

Solution



Consider an element of rod of length 'dx' at a distance x from the point charge q. treating the element as point charge, the force between q and the charge element

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{q\,dQ}{x^2}$$

But dQ = $\frac{Q}{L}dx$ so

∴

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\int_d^{(d+L)}\frac{dx}{x^2}$$

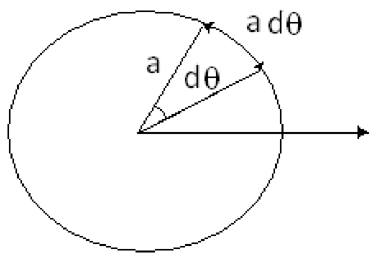
$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\left[\frac{-1}{x}\right]_d^{(d+L)}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\left[\frac{1}{d} - \frac{1}{d+L}\right]$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L(d+L)}$$

Q) A circle, having radius 'a' has liner charge distribution over its circumference having linear charge density $\lambda = \lambda_0 \cos^2\theta$. Calculate the total charge on the circumference of the circle

Solution . The length of an infinitesimally small line element have charge dq = λdl .



But dl = a (dθ)

$dq = a\lambda_0 \cos^2\theta.\ d\theta$

In order to calculate the total charge Q residing on the surface, we have to integrate dq over the entire surface Q = $\oint dq$ here symbol $\oint$ indicates the integer over the entire close path (circumference of the circle)

Q = $\oint a\lambda_0 \cos^2\theta.\ d\theta = a\lambda_0 \int_0^{2\pi} \cos^2\theta.\ d\theta = a\lambda_0\pi$

QUESTIONS (B)

Q) Three equal charges each of magnitude of $2.0X10^{-6}$C are placed at the three corners of right angled triangle of sides 3cm, 4am 5cm. Find the force on the charge at the right angle corner and direction of force [ANs (-22.5,40)N and tanθ = -1.777]

Q) Two electric charges having magnitude 8.0 μC and -2.0μC are separated by 230cm. Where should a third charge be placed so that the resultant force acting on it is zero [20 cm from -2.0μC]

Q) Two spheres having same radius and mass re suspended by two strings of equal length from the same point, in such a way that their surfaces touches each other. On depositing $4X10^{-7}$C charge on them, they repel each other in such a way that in equilibrium the angle between their string becomes 60^{O}. If the distance from the point of suspension to the centre of the sphere is 20cm, find the mass of each sphere g = 10 m/s^2 [Ans : $1.56X10^{-3}$kg]

Q) Three identical charges q are placed on the vertices of n equilateral triangle . Find resultant force acting on the charge 2q kept at its centroid. (distance of centroid from vertices is 1m)

Q) Two identically charged spheres are suspended by stings of equal length. When they are suspended in kerosene, the angle between their strings remains the same as it was in the air. Find the density of the spheres. The dielectric constant of kerosene is 2 and its density is 800 kg m^{-3} [Ans 1600 kg m^{-3}]

Q) If $q_1q_2 > 0$, which type of the force acting between the charges

Q) Two large conducting spheres carrying charges q_1 and q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|q_1||q_2|}{r^2}$$

{Hint: No, explain on the basis of induction value of q_1 and q_2 will alter]

Q) State Coulomb's law in electrostatics.

Q) Force between two point electric charges kept at distance d apart in air is F. If these charges are kept at the same distance in water, how does the force between them charge? Given dielectric constant of water = 80

Q) Dielectric constant of water is 80. What is its permittivity

Q) Define unit of electric charge in terms of electric force

Q) State the principle of superposition of forces in electrostatics

Q) Given two point charges q_1 and q_2 such that q_1 q_2 = <0. What is the nature of the force between them?

Q) Consider three charged bodies P, Q and R. If P and Q repel each other and P attracts R, what is the nature of force between Q and R

Q) In Coulombs law, on what factor the value of electrostatic force constant K depends?

Q) Does Coulombs law of electric force obeys Newton's third law of motion?

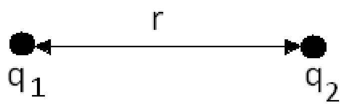
Q) Is the Coulomb force that one charge exerts on other changes if other charges are brought nearby?
Q) Two small balls having equal positive charge q coulomb are suspended by two insulating strings of equal length 1 meter from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity. What is the angle between the two strings and the tension in each string?
Q) An ebonite rod held in hand can be charged by rubbing with flannel but a copper rod cannot be charged like this why?
Q) What is the importance of Coulomb law in vector form?
Q) Electrostatic forces are much stronger than gravitational forces. Give one example

SECTION II
ELECTRIC FIELD

An electric field is defined as a region in which there should be a force on a charge brought into that region. Whenever a charge is being placed in an electric field, it experiences a force

Electric fields are usually produced by different types of charged bodies, point charges, charged plates, charged spheres etc

If two point charges are placed as shown in figure, we can describe the force on them in two ways



(i) The charge q_2 is in the electric field of charge q_1. Thus the electric field of charge q_1 exerts force on q_2.

(ii) The charge q_1 is in the electric field of charge q_2. Hence the electric field of charge q_2 exerts a force on q_1

Electric field intensity or Electric field Strength ($\vec{E}$)

The electric field intensity at a point in an electric field is the force experienced by a unit positive charge placed at that point, it is being assumed that the unit charge does not affect the field.

Thus, if a positive test charge q_0 experiences a force $\vec{F}$ at a point in an electric field, then the electric field intensity $\vec{E}$ at a point is given by

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

<u>Important points regarding electric field intensity</u>

(i) It is vector quantity. The direction of the electric field intensity at a point inside the electric field is the direction in which the electric field exerts force on the unit positive charge.

(ii) Direction of electric field due to positive charge is outward while direction of electric field due to negative charge is inward

(iii) Dimensions of electric field intensity E = $[MLT^{-3}A^{-1}]$

S.I. unit of Electric field is C/C or V/m as

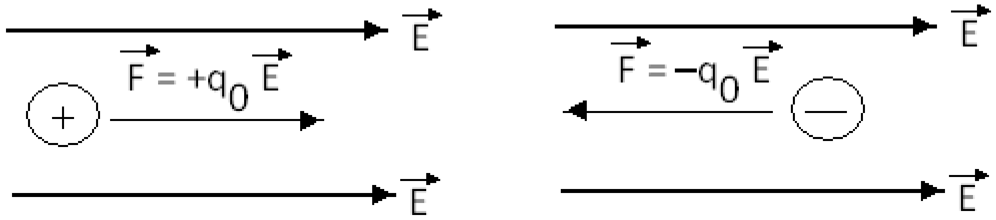
$$\frac{N}{C} = \frac{N \times m}{C \times m} = \frac{J}{C \times m} = \frac{V}{m}$$

Force exerted by a field on a charge inside it

By definition $\vec{E} = \frac{\vec{F}}{q_0}$ or $\vec{F} = q_0\vec{E}$

If q_0 is positive charge force $\vec{F}$ on it is in the direction of $\vec{E}$

If q_0 is negative charge force $\vec{F}$ on it is opposite to the direction of $\vec{E}$



Solved numerical

Q) An electron (q =-e) is placed near a charged body experiences a force in the positive y direction of magnitude 3.6X10^{-8} N

() What is the electric field at that location

(b) What would be the force exerted by the same charged body on an alpha particle (q = +2e) placed at the location initially occupied by the electron

Solution (a)

From equation E = F/q = 3.6X10^{-8}/ 1.6X10^{19} = 2.25X10^{11} N/c

We know electron is negatively charged particle hence will move opposite to the direction of electric field . Given electron experiences for is positive direction, electric field must be in negative direction

(b) The force on alpha particle will be F = Eq = 2.25X10^{11} (2X1.6X10^{19}) = 7.20X10^{-8} N

Since alpha particle is positively charge it will experience force in the direction of electric field

Electric field due to point charge

Let a positive test charge q_0 be placed at a distance r from a point charge q. The magnitude of force acting on q_0 is given by Coulomb's law.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}$$

The magnitude of the electric field at the site of the charge is

$$\boxed{E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

Electric field in vector form is

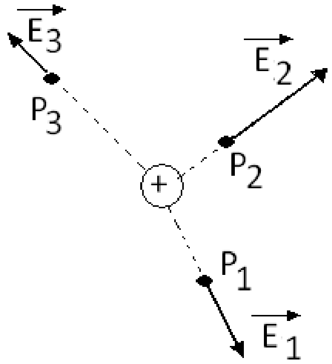
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}}$$

Here $\vec{r}$ is the position vector of point

The direction of $\vec{E}$ is the same s the direction of $\vec{F}$, along a radial line q, point outward if q is positive and inward is q is negative.

The figure given shows the direction of the electric field $\vec{E}$ at various points near a positive point charge.

Note length of arrow is more where electric field is more and point near to charge have more electric field



Electric field intensity due to a group of point charges

Since the principle of linear superposition is valid for Coulomb's law, it is also valid for the electric field. To calculate the electric field at a point due to a group of N point charges. We find the individual field strength $\vec{E}_1$ due to Q_1, $\vec{E}_2$ due to Q_2 and so on.. The resultant field strength is the vector sum of individual field strengths

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

Electric field at a point depends on the charge and position of point

Consider a system of charges q_1, q_2, q_3 , ……q_n with position vectors $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, ---$\vec{r}_n$ relative to some origin O.

Electric field at point P having position vector $\vec{r}$. For this purpose place a very small test charge q_0 at that point and use the superposition principle.

Electric field at point P due to q_1 is given by

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = k\frac{q_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r} - \vec{r}_1)$$

Electric field at point P due to q_2 is given by

$$\vec{E}_2 = \frac{\vec{F}_2}{q_0} = k\frac{q_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2)$$

Electric field at point P due to q_3 is given by

$$\vec{E}_3 = \frac{\vec{F}_3}{q_0} = k\frac{q_3}{|\vec{r} - \vec{r}_3|^3}(\vec{r} - \vec{r}_3)$$

Same way, electric field at point P due to charge q_n is

$$\vec{E}_n = \frac{\vec{F}_n}{q_0} = k\frac{q_n}{|\vec{r} - \vec{r}_n|^3}(\vec{r} - \vec{r}_n)$$

According to superposition principle, net electric field at a point P is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n$$

$$\vec{E} = k\frac{q_1}{|\vec{r} - \vec{r}_1|^3}(\vec{r} - \vec{r}_1) + k\frac{q_2}{|\vec{r} - \vec{r}_2|^3}(\vec{r} - \vec{r}_2) + \cdots + k\frac{q_n}{|\vec{r} - \vec{r}_n|^3}(\vec{r} - \vec{r}_n)$$

$$\vec{E} = k\sum_{j=1}^{n} \frac{q_j}{|\vec{r} - \vec{r}_j|^3}(\vec{r} - \vec{r}_j)$$

Here q_1, q_2, q_3 ,…. Are the sources of electric field

Physical significance of electric field

a) Equation for electric field is given by

$$\vec{E} = k\sum_{j=1}^{n} \frac{q_j}{|\vec{r} - \vec{r}_j|^3}(\vec{r} - \vec{r}_j)$$

Equation of force acting on a unit positive charges at point $\vec{r}_{(x,y.z)}$ once $\vec{E}_{\vec{r}}$ is kown, we do not have to worry about the source of electric field. In this since, the electric field itself is a special representation of the system of charges producing electric field, as far as the effect on other charges are concerned. Once the representation is done, the force acting on charge q kept at that point in the electric field can be determined using following equation

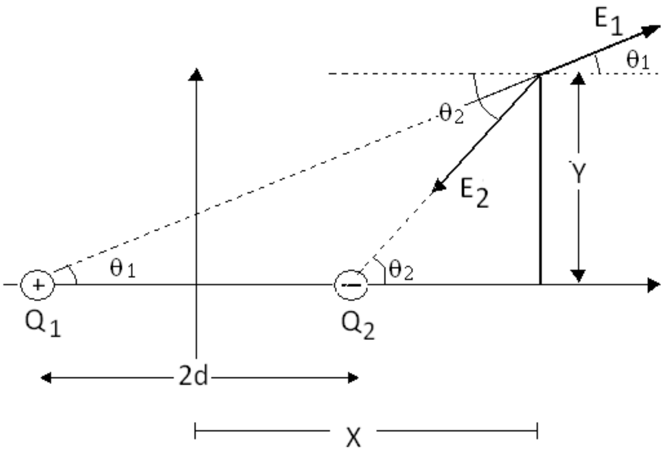
$$\vec{F}_{\vec{r}} = q\vec{E}_{\vec{r}}$$

b) True significance of electric field is when there is an accelerated motion of charge q_1 and q_2. For effect of q_1 and q_2 there will be a time delay between the force on q_2 and the cause (motion of q_1) .

The field picture is this: the accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed of light and reaches q_2 and cause a force on q_2. The notation of field elegantly accounts for the time delay. Thus even though electric and magnetic fields can be detected only by their effect (force) on charge, they are regarded as physical entity. Electric and magnetic field transport energy. Thus, a source of time-dependent electromagnetic fields, turned on briefly and switch off, leaves behind propagating electromagnetic field transporting energy

Solved numerical

Q) A point charge Q_1 = 20μC is at (-d,0) while Q_2 = 10 μC is at (+d, 0). Find the resultant field strength at a point with coordinates (x, y). Take d = 1.0m and x=y=2m



Solution

From figure we have

$r_1 = \sqrt{(x^2+y^2)^2+y^2}$ = √13 = 3.6 m

$r_2 = \sqrt{(x^2-y^2)^2+y^2}$ = √5 = 2.2 m

The magnitudes of the fields are

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1|}{r_1^2} = \frac{(9.0\times10^9)(2\times10^{-5})}{13} = 1.385\text{X}10^4 \text{ N/C}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|Q_2|}{r_2^2} = \frac{(9.0\times10^9)(10^{-5})}{5} = 1.8\text{X}10^4 \text{ N/C}$$

The components of the resultant field strength are

$E_x = E_{1x} + E_{2x} = E_1\cos\theta_1 - E_2\cos\theta_2$

$E_y = E_{1y} + E_{2y} = E_1\sin\theta_1 - E_2\sin\theta_2$

From figure $\sin\theta_1 = y/r_1$; $\sin\theta_2 = y/r_2$; $\cos\theta_1 = (x+d)/r_1$; $\cos\theta_2 = (x-d)/r_2$

Therefore

$$E_x = (1.385\times10^4)\frac{3}{3.6} - (1.8\times10^4)\frac{1.0}{2.2} = 3.32\times10^3\text{N/C}$$

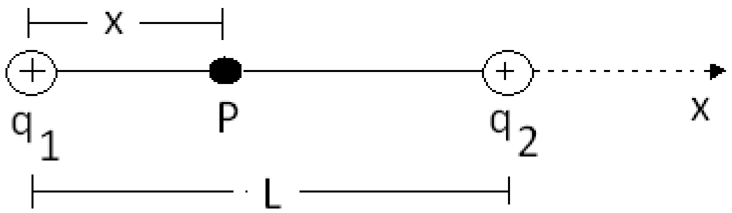
$$E_y = (1.385\times10^4)\frac{2}{3.6} - (1.8\times10^4)\frac{2.0}{2.2} = -8.66\times10^3\text{N/C}$$

Hence

$$\vec{E} = (3.32\times10^3)\hat{\imath} - (8.66\times10^3)\hat{\jmath} \text{ N/C}$$

Q) A point charge q_1 of +1.5 μC is placed at a origin of a coordinate system, and the second charge q_2 of +2.3 μC is at position x = L, where L = 13 cm. At what point P along the x-axis is the electric field zero

Solution



The point must lie between the charges because only in this region the force exerted by q_1 and q_2 on a test charge oppose each other. If $\vec{E}_1$ is the electric field due to q_1 and $\vec{E}_2$ is that due to q_2, the magnitudes of these vectors must be equal to or $E_1 = E_2$

We have

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(L-x)^2}$$

Where x is the coordinate of point P

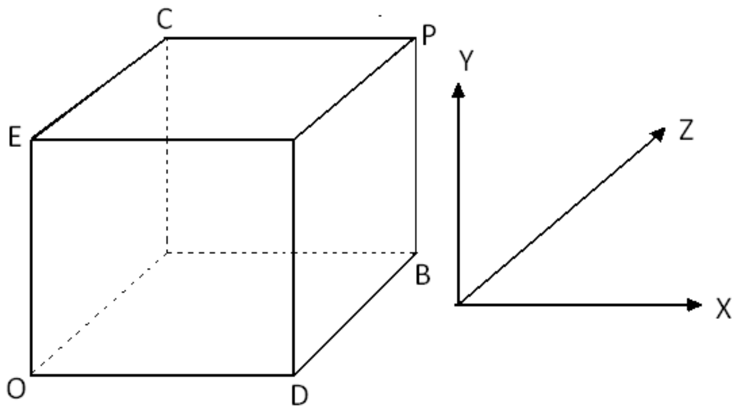
On solving we get $x = \frac{L}{1\mp\sqrt{q_1/q_2}}$ substituting the values we get

X = 5.8 cm and -54.8 cm

But the negative value of x is unacceptable

Hence x = 5.8cm

Q) A cube of edge 'a' carries a point charge q at each corner. Calculate the resultant force on any one of the charges



Solution :

Let us take one corner of the cube as origin O(0, 0, 0) and the opposite corner P as (a,a,a). We will calculate the electric field at P due to the other seven charges at corners.

Expressing the field of a point charge in vector form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$$

Coordinates of A = (a,a,0)
Coordinates of B = (a,0,a)
Coordinates of C = (0,a,a)
Coordinates of D = (a,0,0)
Coordinates of E = (0,a,0)
Coordinates of F = (0,0,a)

Electric field strength at P due to charges A, B, C

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^3}\left[\overrightarrow{AP} + \overrightarrow{BP} + \overrightarrow{CP}\right]$$

$\overrightarrow{AP}$ = P - A = (a,a,a) – (a,a,0) = (0,0,a) and $|\overrightarrow{AP}|$ = a
$\overrightarrow{BP}$ = P-B = (a,a,a) – (a,0,a) = (0,a,0) and $|\overrightarrow{BP}|$ = a
$\overrightarrow{CP}$ = P-C= (a,a,a) – (0,a,a) = (a,0,0) and $|\overrightarrow{CP}|$ = a

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^3}\left[\hat{\imath} + \hat{\jmath} + \hat{k}\right]$$

Electric field strength at P due to charges at D, E , F

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^3}\left[\overrightarrow{DP} + \overrightarrow{EP} + \overrightarrow{FP}\right]$$

$\overrightarrow{DP}$ = (a,a,a) – (a,0,0) = (0,a,a) and $|\overrightarrow{DP}|$ = a√2
$\overrightarrow{EP}$ = (a,a,a) – (0,a,0) = (a,0,a) and $|\overrightarrow{EP}|$ = a√2
$\overrightarrow{FP}$ = (a,a,a) – (0,0,a) = (a,a,0) and $|\overrightarrow{EP}|$ = a√2

$$\left[\overrightarrow{DP} + \overrightarrow{EP} + \overrightarrow{FP}\right] = (0, \mathrm{a}, \mathrm{a}) + (\mathrm{a}, 0, \mathrm{a}) + (\mathrm{a}, \mathrm{a}, 0)$$

$$\left[\overrightarrow{DP} + \overrightarrow{EP} + \overrightarrow{FP}\right] = (2\mathrm{a}, 2\mathrm{a}, 2\mathrm{a})$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(a\sqrt{2}\right)^3}\left[2a\hat{\imath} + 2a\hat{\jmath} + 2a\hat{k}\right]$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\sqrt{2}a^2\right)}\left[\hat{\imath} + \hat{\jmath} + \hat{k}\right]$$

Electric field strength at point P due to O

$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(a\sqrt{3})^3}\left[\overrightarrow{OP}\right]$ (We have OP = a√3]

$\overrightarrow{OP}$ = = (a,a,a) – (0,0,0) =(a,a,a)

Hence resultant electric field at P

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$\vec{E} = \frac{q[\hat{\imath}+\hat{\jmath}+\hat{k}]}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$ outward along OP

Force on the charge at P is F = qE

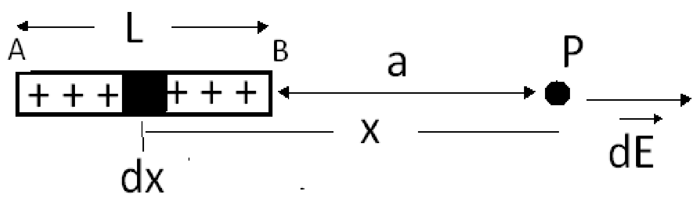
$\vec{F} = \frac{q^2[\hat{\imath}+\hat{\jmath}+\hat{k}]}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$ outward along OP

Magnitude

$F\ \frac{q\sqrt{3}}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$ outward along OP

Q) A thin rod of length L carries a uniformly distributed charge Q. Find the electric field strength at a point along its axis at a distance ‘a’ from one end

Solution:



Let us consider an infinitesimal element of length d at a distance x from the point P. The charge on this element is dq = λdx. Where λ = Q/L is the linear charge density.

The magnitude of the electric field at P due to this element is

$\mathrm{dE} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$ and its direction is to the right since λ is positive. The total electric field strength E is given by

$$\mathrm{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{a+L}\frac{dx}{x^2}$$

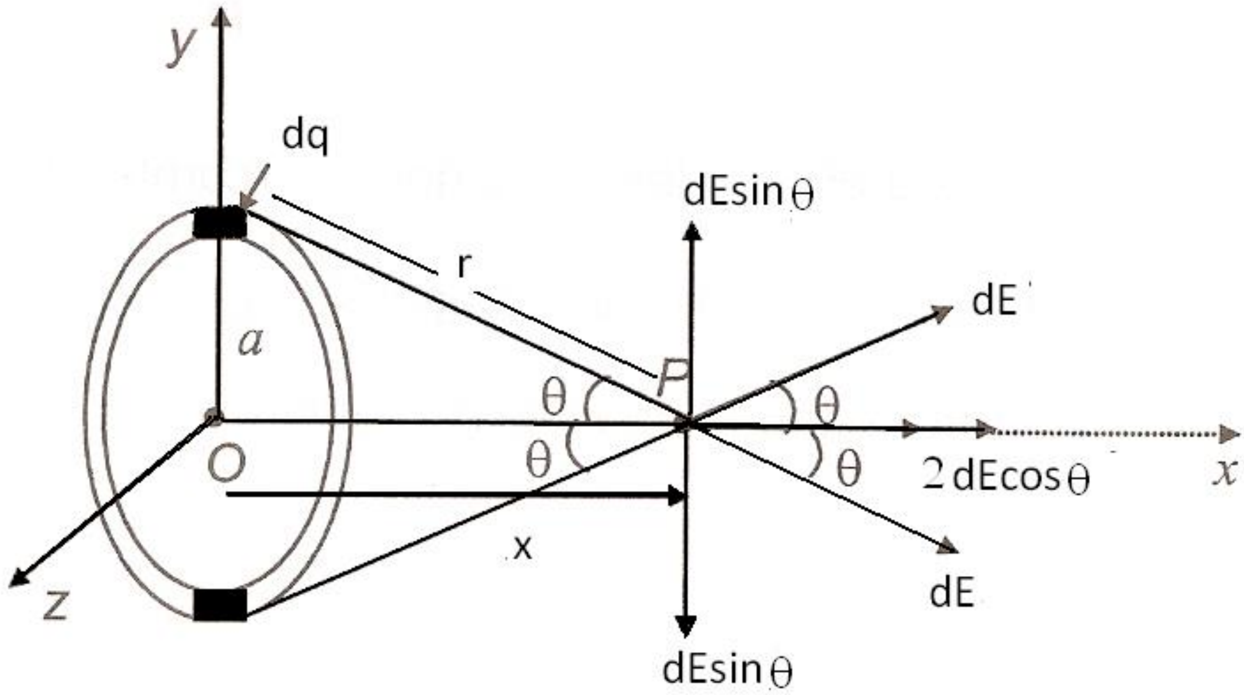
$$\mathrm{E} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_a^{a+L}$$

$$\mathrm{E} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{a+L}\right]$$

But Q = λL

$$\mathrm{E} = \frac{Q}{(4\pi\varepsilon_0)a(a+L)}$$

Electric field due to a uniformly charged ring at a point on the axis of the ring



Let us consider a charge Q distributed uniformly on a thin, circular, non-conducting ring of radius a. We have to find electric field E at the point P on the axis of the ring, at a distance x from the centre.

From symmetry we observe that every element dq be paired with a similar element on the opposite side of the ring. Every component dEsinθ perpendicular to the x-axis is thus cancelled by a component dEsinθ in the opposite direction. In a summation process, all the perpendicular components add to zero. Thus we only add dE_X components

Now $r^2 = a^2 + x^2$ and $\cos\theta = \frac{x}{\sqrt{a^2+x^2}}$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(d^2+x^2)}$$

Hence, the resultant electric field at P is given by

$$E = \int dE_X = \int dE cos\theta$$

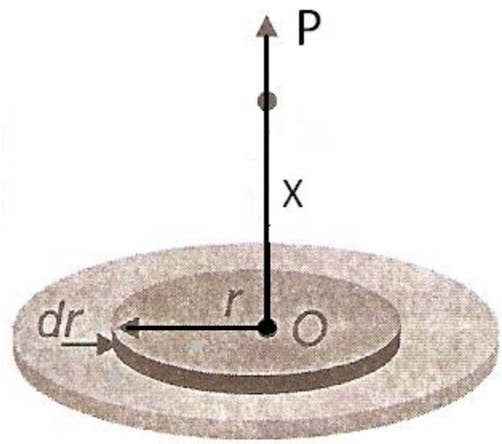
$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{(d^2+x^2)}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{x}{(d^2+x^2)^{3/2}}\int dq$$

As we integrate around the ring, all the terms remain constant and $\int dq = q$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(d^2+x^2)^{3/2}}}$$

Electric field due to a uniformly charged disc at a point on the axis of the disc



Let us consider a flat, circular, non-conducting thin disc of radius *R* having a uniform surface charge density σ C/m^2. We have to find the electric field intensity at an axial point at a distance *x* from the disc.

Let O be the centre of a uniformly charged disc of radius R and surface charge density σ. Let P be an axial point, distance x from O, at which electric field intensity is required.

From the circular symmetry of the disc, we imagine the disc to be made up of large number of concentric rings and consider one such ring of radius 'r' and infinitesimally small width dr.

The area of the elemental ring = Circumference X width = (2πr dr)

The charge dq on the elemental ring = (2πr dr) σ

Therefore, the electric field intensity at P due to the elementary ring is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{x(2\pi r\, dr)\,\sigma}{(r^2+x^2)^{3/2}}$$

And is directed along x –axis. Hence, the electric intensity E due to the whole disc is give by

$$E = \frac{x\sigma}{2\varepsilon_0}\int_0^R \frac{r\, dr}{(r^2+x^2)^{3/2}}$$

$$E = \frac{x\sigma}{2\varepsilon_0}\left[\frac{-1}{(r^2+x^2)^{1/2}}\right]_0^R$$

$$E = \frac{x\sigma}{2\varepsilon_0}\left[\frac{-1}{(R^2+x^2)^{1/2}}+\frac{1}{x}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{(R^2+x^2)^{1/2}}\right]$$

Electric lines of Forces

The properties of electric lens of forces are the following

(i) The electric lines of force are continuous curves in an electric field starting from a positively charged body and ending on a negatively charged body.

(ii) The tangent to the curve at any point gives the direction of the electric field intensity at that point

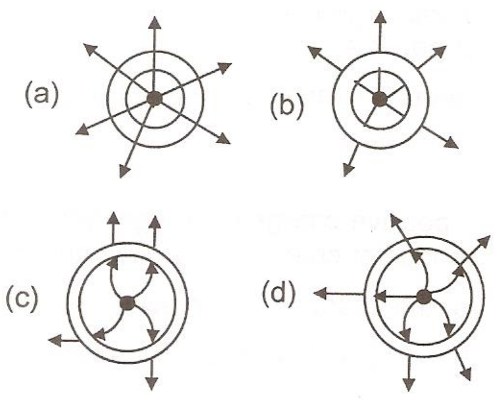
(iii) Electric field lines of forces do not pass but leave or end on a charged conductor normally.

Suppose the line of forces is not perpendicular to the conductor surface. In this situation, the component of electric field parallel to the surface cause the electrons to move and hence conductor will not remain equipotential which is absurd as electrostatics conductor is an equipotential surface.

(iv) Electric field line of forces never intersects since if they cross at a point, electric field intensity at that point will have two directions, which is not possible.

(v) The number of electric lines of force that originate from or terminate on a charge is proportional to the magnitude of the charge.

(vi) As number of lines of force per unit area normal to the area at point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field. Further, if the lines of force are equidistant straight lines, the field is uniform.



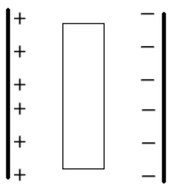
Solved numerical

Q) A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric field lines of force

Solution:

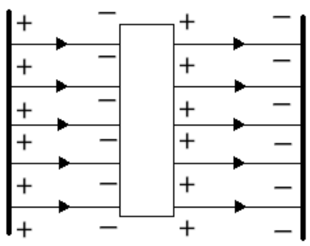
Option B is correct because it satisfy following property (i) Electric field lines due to point charge is radial (ii) Electric field lines of forces do not pass but leave or end on a charged conductor normally.

Q) A metallic slab is introduced between the two charged parallel plates as shown in figure. Sketch the electric field lines of force between the plates



Solution: Keeping in mind that

(i) Electric lines of force starts from positive charge and end on negative charge



(ii) Electric lines of force start and end normally on the surface of a conductor

(iii) Electric lines of force do not exist inside a conductor.

QUESTIONS (C)

Q) A thin glass rod is bent into a semicircle of radius r. A charge +q is distributed over it. Find the electric field E at a centre of the semicircle

Q) A charged $+10^{-9}$C is located at the origin of a Cartesian co-ordinate system and another charge Q at (2, 0, 0) m. IF X-component of electric field at (3,1,1)m is zero calculate the value of Q [Ans -0.43×10^{-9}C]

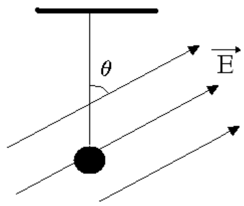
Q) Four electric charges +q, +q, -q and –q are respectively placed on the vertices A, B, C and D of a square. The length of the square is 'a', calculate the intensity of the resultant electric field at the centre[Ans E = $\frac{4\sqrt{2}kq}{a^2}$

Q) Four particles. each of charge q is placed on the four vertices of a regular pentagon. The distance of each corner from the centre is 'a'. Find the electric field at the centre of the pentagon [E = kq/a^2]

Q) An electron falls through a distance of 1.5cm in a space, devoid of gravity, having uniform electric field of intensity $2.0X10^4$ N/C. The direction of electric field intensity is then reversed keeping its magnitude same, in which a proton falls through the same distance. Calculate the time taken by both of them $m_e = 9.1X10^{-31}$kg and $m_P = 1.7X10^{-27}$kg and $e = 1.6X10^{-19}$C

Q) An arc of radius r subtends an angle θ at the centre with the x-axis in a Cartesian coordinate system. A charge is distributed over the arc such that the linear charge density is λ. Calculate the electric field at the region. Ans $E = \frac{K\lambda}{r}[(-sin\theta)\hat{\imath} + (cos\theta - 1)\hat{\jmath}]$

Q) A simple pendulum is suspended in a uniform electric field $\vec{E}$ as shown in figure. What will be its period if its length is l? Charge on the bob of pendulum is q and mass m



Ans. $T = 2\pi\sqrt{\frac{l}{\left((g)^2+\frac{q^2E^2}{m^2}-\frac{2gqE}{m}cos\theta\right)^{1/2}}}$

Q) What is test charge? What should be its magnitude
Q) A charged particle is fired with velocity $\vec{v}$ making a certain angle with an electric line of force. Will the charged particle move along the line of force?
Q) A small test charge is relased at rest at a apoint in an electrostatic field configuration. Will it travel along the line of force passing through that point?
Q) What is the relation between electric field strength and force?
Q) Draw line of force to represent a uniform electric field
Q) Define electric field at a point
Q) What electric field lines represent?
Q) Define dielectric constant of a medium in terms of force between two electric harges
Q) What is the dielectric constant of metal ? [Ans infinity]
Q) Name the physical quanity whose SI unit is N/C
Q) Four charges of same magnitude and same sign are placed at the corners of a square, of each side 0.1m. What is electric field intensity at the centre of the square?
Q) Name any for vector field? [Ans. Electric , Magnetic , gravitational, flow fild of flude]
Q) Force experenced by an electron in an electric field $\vec{E}$ is F newton. What will be the force experienced by a proton in the same field? Take mass of proton to be 1836 times the mass of electron

Q) Define electric field intensity at a point

Q) Two point charges of +3μC each are 100cm apart. At what point on the line joining the charges will the electric intensity be zero?

Q) How does a free electron at rest move in an electric field?

Q) A plastic comb run through one's hair attracts small bits of paper. Why? What happens if hair is wet or if it is raining?

Q A charge dparticle is free to move in an electric field. Will it always move along an electric line of force?

Q) Give two propertties of electric field lines of force. Sketch them for an isolated positive charge.

Q) Sketch the electric field lines of force due to point charge +q and –q

Q) Two point charges of unkown magnitued and sign are palced some distance apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential condition for this to happen

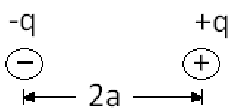
Q) What is an electric line of force? What is its importance?

Q) A block of mass m carrying charge q is placed on a frictionless horizontal surface. The block is connected to a rigid wall through an unstressed spring of spring constant k. A horizontal uniform electric field E parallel to the spring is swithched on,. Find the amplitude of the sresulting simple harmonic motion of the block [ANs a = qE/k]

Q) A metal sphere is held fixed on a smooth horizontal insulated plate and another metal sphere is palced some distance away. If the fixed sphere is given a charge , how will other sphere react?

Electric Dipole

A system of equal and opposite charges, separated by a finite distance is called as an electric dipole.



As shown in figure, the two electric charges of electric dipole are +q and –q and distance between them is 2a. Electric dipole moment ($\vec{P}$) of the system can be defined as follows

$$\boxed{\vec{P} = q(2\vec{a})}$$

Importatnt points regarding electric dipole

(i) The SI unit of electric dipole is coulomb metre (C m)

(ii) Electric dipole is a vector quantity and its direction is from negative charge (-q) to positive charge (+q)

(iii)The net electric charge on an electric dipole is zero but its electric field is not zero, since the position of the two charges is different .

If lim q→ ∞ and a→ 0 in $\vec{P} = q(2\vec{a})$, then electric dipole is called point dipole.

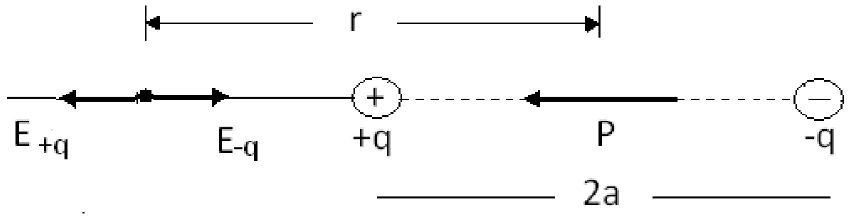
Electric field of a dipole

The electric field of the pair of charges (-q and +q) at any point in space can be found out from Coulombs law and the superposition principle.

The resultants are simple for the following two cases:
(i) When the point is on the dipole axis
(ii) When it is in the equatorial plane

(i) Electric field due to dipole for point on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q as shown in figure, then



Elctric field due to negative charge at point P is given by

$$\vec{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{p}$$

where $\hat{p}$ is the unit vector along the dipole axis (from –q to +q)
Electric field at point P due to positive charge is given by

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p}$$

Total electric field at P $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]\hat{p}$$

But 2qa = P

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2pr}{(r^2-a^2)^2}\right]\hat{p}}$$

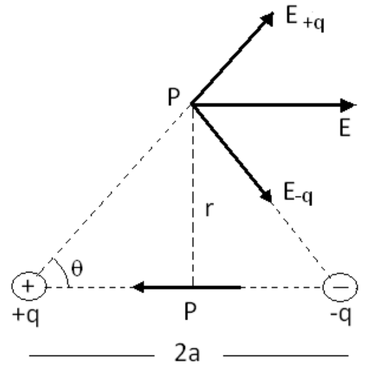
For r >> a

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]\hat{p}}$$

Direction of electric field is in the direction of electric dipole moment

(ii) Electric field due to dipole for point on the equatorial plane

Let the point P be at distance r from the centre of the dipole on the equatorial plane as shown in figure. Then



Magnitude of electric field at point P due to positive charge is given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

Magnitude of electric field at point P due to negative charge is given by

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

Both magnitude are equal . Directions of $\vec{E}_{-q}$ and $\vec{E}_{+q}$ are as shown in figure .

Clearly componants perpendicular to axis cancel away. The componant along the dipole axis add up. The total electric field is opposite to $\hat{p}$. We have

$\vec{E} = -(E_{+q} + E_{-q})cos\theta\hat{p}$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{2}{r^2 + a^2}cos\theta\hat{p}$$

From figure

$$cos\theta = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{2}{r^2 + a^2}\frac{a}{\sqrt{r^2 + a^2}}\hat{p}$$

$$\boxed{\vec{E} = \frac{2qa}{4\pi\varepsilon_0(r^2 + a^2)^{3/2}}\hat{p}}$$

At large distance (r>>a), this reduces to

$$\vec{E} = \frac{2qa}{4\pi\varepsilon_0(r)^3}\hat{p}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{p}{(r)^3}\right]\hat{p}}$$

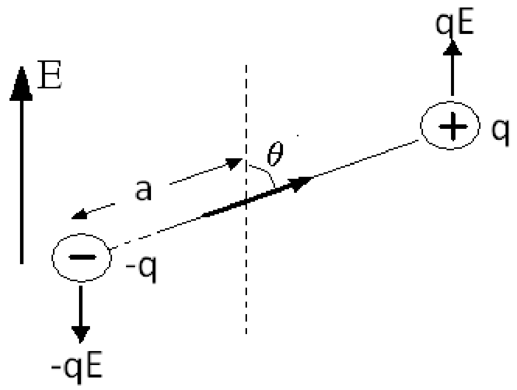
Important points

(i) The electric field at large distances falls of as $1/r^3$

(ii) The magnitude and direction of the dipole field depends not only on the distance r but also on the angle between the position vector $\vec{r}$ and dipole moment $\vec{p}$

(iii) Electric field of dipole is cylindrically symmetrical.

Dipole in uniform external electric field



As shown in figure an electric dipole of magnitude p = q(2a) is kept in a uniform electric field . Let θ be the angle between dipole momentum $\vec{p}$ and electric field $\vec{E}$.

The force q$\vec{E}$ and -q$\vec{E}$ are acting on the charges q and –q respectively. These forces are equal but opposite in direction.

The resultant force being zero keeps dipole in translational equilibrium. But, the two forces have different line of action, hence the dipole will experience a torque.

When the net force is zero, the torque (couple) is independent of the origin. Its magnitude is equal to the magnitude of the force multiplied by the arm of the couple (perpendicular distance between the two antiparallel forces)

Magnitude of torque = qEX2asinθ = 2qaEsinθ but $q(2a) = p$

$$\boxed{\text{Thus} \qquad \vec{\tau} = \vec{p} \times \vec{E}}$$

Direction of torque is perpendicular to paper, coming out.

The torque rotates the dipole in such a way that the angle θ reduces,

(i) when the dipole aligns itself along the direction of electric field, the torque becomes zero. This is the normal position of dipole in electric field.

(ii)If the dipole is to be rotated by an angle θ from this position, work is required to be done against torque. This work is equal to the change in potential energy of dipole.

Behaviour of electric dipole in non-uniform electric field

If the electric field is not uniform, the intensity of electric field will be different at different points as a result electric force acting on the positive charge and negative charge of the dipole will also be different. In this situation the net force and torque are acting on the dipole. As a result dipole experiences a linear displacement in addition to roatation. This rotation will continue only till the dipole aligns in the direction of the electric field. But linear motion will continue.

When charged comb is brough near the piece of parer, the non-uniform electric field is produced by the comb. Electric dipole is induced along the direction of non-uniform eletric field in small piece of paper. Now non uniform electric field exters a net force on piece of paer and paper moves in the direction of comb.

QUESTIONS (D)

Q) Define the elctric dipole moment and give SI unit

Q) What will be the torque acting on the dipole, if ut is palced parallel to the electric field.

Q) Why bits of paper gets attarcted to charged comb

Q) Why does ythe two electric field lines do not intersecting each other?

Q) Draw the electric field lines of electric dipole

Q) What is the direction of elecric dipole moment vector of an electric field

Q) What is an ideal dipole

Q) Define electricdipole

Q) Two equal and opposite point charges are separeated by a certain distance. What are the points at which the resultant electric field is parallel to the line joining two charges

Q) How does the torque affact the dipole in an electric field

Q) What is the net force on an elelectric dipole palced in a uniform electric field?

Q) Is torque on the an electric dipole a vector or scalar?

Q) Give SI unit of electric moment of dipole

Q) When is the elelctric dipole in ustable equilibrium

Q) What is meant by the statement that the electric field of a point charge has spherical symmetry whereas that of an electric dipole is cylindrically symmetrical?

Q) What is the orientation of an electric dipole ina uniform electric field corresponding to stable equilibrium?

Flux of an electric field or Electric flux

Let us consider a plane surface of area S placed in an electric fied $\vec{E}$.

Electric flux through an elementary area $\overrightarrow{dS}$ is defind as the scalar product of $\overrightarrow{dS}$ and $\vec{E}$ .i.e

d $\phi_E = \vec{E} \cdot \overrightarrow{dS}$, where $\overrightarrow{dS}$ is the area vector, whose magnitude is the area dS of the element and whose direction is along the outward normal to the elementary area.

Hence, the electric flux through the entire surface is given by

$\phi_E = \int \vec{E} \cdot \overrightarrow{dS}$ or $\phi_E = \int E\, dS \cos\theta$

θ is the angle between area vector and electic field

Area vector is always outward for closed surface. While for other surface it can be considered as inward outward.

If the electric field is uniform, then $\phi_E = E \cos\theta \int dS$

When the electric flux through a closed surface is required, we use a small circular sign on the integration symbol

$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS}$

Thus general defination of electic flux can be given as " The flux linked with any surface is the surface integration of the electric field over the given surface"

Unit of flux N-m/C

Importatnt points regarding electric flux

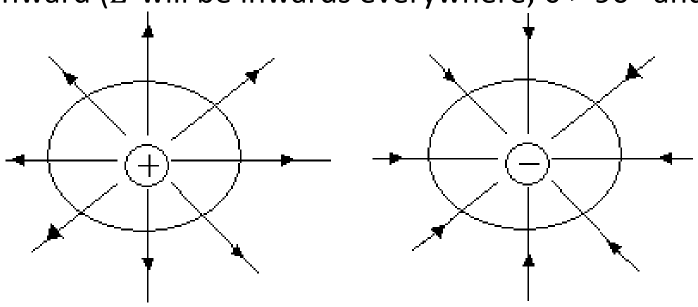
(i) The number of lines of force passing normally to the given area gives the measure of flux of electric field over the given area.

(ii) It is a real scalar physical quantity with units (volt xm)

(iii) It will be maximum when cosθ = max = 1. i.e θ = 0^O, i.e. electric field is normal to the surface with $(d\,\phi_E)_{max} = E(dS)$

(iv) It will be minimum when |cosθ| = min = 0 , θ = 90^O i.e. field parallel the area with $(d\,\phi_E)_{min} = 0$

(v) For closed surface, ϕ_E is positive if the lines of force point outward everywhere ($\vec{E}$ will be outward everywhere, θ < 90^O and $\vec{E} \cdot \overrightarrow{dS}$ will be positive) and negative if they point inward ($\vec{E}$ will be inwards everywhere, θ > 90^O and $\vec{E} \cdot \overrightarrow{dS}$ will be negative)



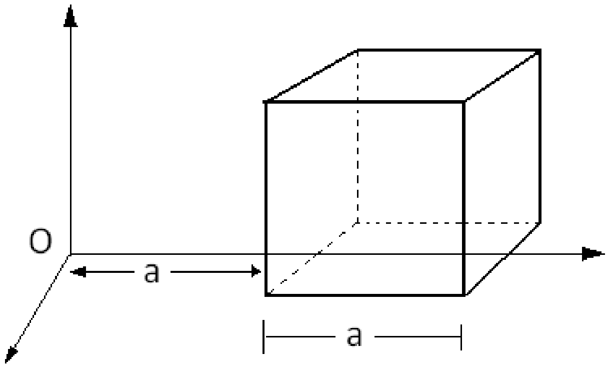
Positive flux $\phi > 0$ Negative flux $\phi < 0$

Solved numerical

Q) In a region of space the electric field is given by $\vec{E} = 8\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$. Calculate the eletrci flux through a csurface of area 100 units in x-y plane

Solution: Since surface is in x-y plane area vector is along z direction thus $\vec{S}$=100 $\hat{k}$. Electric flux $\phi_E = \vec{E}.\vec{S} = (8\hat{\imath} + 4\hat{\jmath} + 3\hat{k})(100\hat{k}.)$ = 300 units

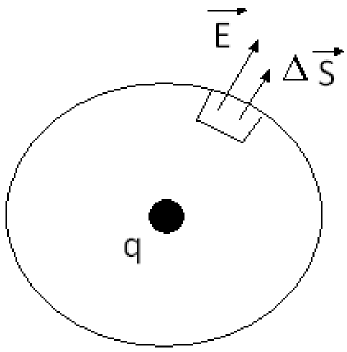
Q) Calculate the electric flux through a cube of side 'a' as shown, where $E_x = bx^{1/2}$, $E_Y = E_z$, =0 , a = 10cm and b = 800 N/C-m$^{1/2}$



Solution:
The electric field throughout the region is no-uniform and its x-component is given by $E_x = bx^{1/2}$.
Now for the left face peroppendicular to x-axis, we have x = a = 10cm,
Electric field for left face $E_X = 800(10)^{1/2}$
Hence flux for left face = $-E_X(a^2)$ Here sign is negative as flux is negative

while for the right face x = 2a = 20 cm
Electric field for left face $E'_X = 800(20)^{1/2}$
Hence flux for left face = $E'_X(a)^2$ Here sign is positive as flux is positive
Flux through remaining faces is zero as electric field is zero
thus total flux passing through a cube = $E'_X(a)^2 - E_X(a^2) = a^2 [E'_X - E_X] = [(20)^{1/2}-(10)^{1/2}](10X10^{-2})^2$ = 1.05 N-m/C

Q) A charge q is placed at the centre of a sphere. Find the flux of the electric field through the surface of the sphere due to the enclosed charge
Solution



Let us take a small element ΔS on the surface of the sphere. The electric field here is radially outward and has the magnitude Kq/r^2 , where r is the radius of the sphere
The electric flux through this element is

$\Delta \phi_E = \vec{E}.\Delta\vec{S} = \frac{Kq}{r^2}\Delta S$ (as $\theta = 0^O$)

Hence electric flux through entire sphere is given by

$\phi_E = \sum\Delta \phi_E = \frac{q}{4\pi\varepsilon_0}\sum \Delta S$

$\phi_E = \frac{q}{4\pi\varepsilon_0}(4\pi r^2) = \frac{q}{\varepsilon_0}$

Gauss's Law

This law gives a relation between the electric flux through any closed hypothetical surface (called gaussian surface) and the charge enclosed by the surface. It states ***"The electric flux (ϕ_E) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the /net' charge enclosed by the surface"*** That is

$\phi_E = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{\Sigma q}{\varepsilon_0}$ --(i)

Where $\sum q$ denotes the algebric sum of all the charges enclosed by the surface.

If there are several charges $+q_1$, $+q_2$, $+q_3$, $-q_4$, $-q_5$ …etc inside the Gaussian surface then

$$\sum q = q_1 + q_2 + q_3 - q_4 - q_5 \ldots..$$

It is clear form above equation that flux inked with a closed body is independent of the sphape and size of the body and position of charge inside it

The law implies that the total elecric flux through a closed surface is zero, if no net charge is enclosed by the surface

Important points regarding law

(i) Gaus's law is true for any closed surface, no matter what its shape or size

(ii) The term q on the right side of the eqation , include the sum of all charges enclosed by the surface. The charges may be located any where inside the surface

(iii) The electric field appearing on the left hand side of the equation(1) is the electric field produced due to a system of charges, whether enclosed by the surface or outside it

(iv) The surface that we choose for application of Gauss's law is called Gaussian surface

(v) Gauss law is useful towards a much easier calculation of electric field when system has some symmetry.

Applications of Gauss's Law

Gauss's law is useful when there is symmetry in the charge distribution, as in the case of uniformly charged sphere, long cylinders and flat surface over which the surface integral gives by equation (1) can be easily evaluated

These are steps to apply the Gauss's law

(i) Use symmetry of the charge distribution to determine the pattern of the lines

(ii) Choose a Gaussian surface for which $\vec{E}$ is either parallel to $\overrightarrow{dS}$ or perpendicular to $\overrightarrow{dS}$

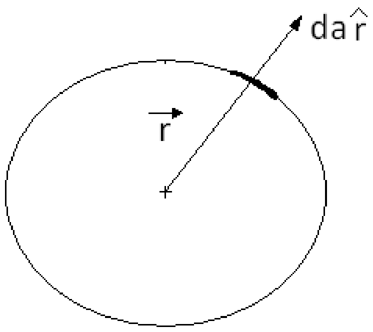
(iii) If $\vec{E}$ is parallel to $\overrightarrow{dS}$, then the magnitude of $\vec{E}$ should be constant over this part of the surface.

The integral then reduces to sum over area elements.

Solved numerical

Q) An electric field prevailing in a region depends on x and y co-ordinates according to an equation $\vec{E} = b\,\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$ where b is a constant. Find the flux passing through a sphere of radius r whose centre is on the origin of the co-ordinate system.

Solution



As shown in figure, $\hat{r}$ is the unit vector in the direction of $\vec{r}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r}$$

Now $\vec{E} = b\,\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$

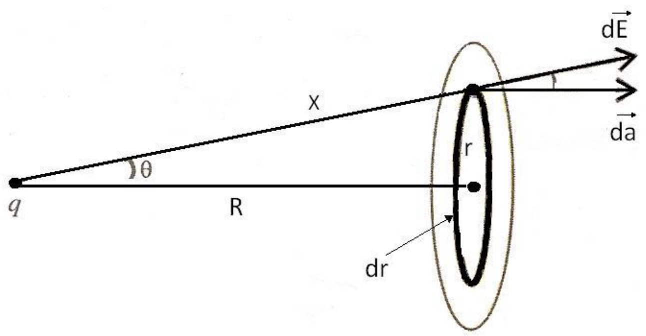
$$\therefore \vec{E}.\overrightarrow{da} = b\left(\frac{x\hat{i}+y\hat{j}}{x^2+y^2}\right).\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}da$$

$$\vec{E}.\overrightarrow{da} = \frac{b\ da}{r}\frac{x^2+y^2}{x^2+y^2} = \frac{b}{r}da$$

$$\int \vec{E}.\overrightarrow{da} = \frac{b}{r}\int da = \frac{b}{r}4\pi r^2 = 4\pi r$$

Q) Calculate the total electric flux linked with a circular disc of radius a. situated at a distance R from a point charge q

Solution



Consider a thin circular ring of radius r and dr as shown in figure. The electric field intensity at some point P on ring is given by.

$$\left|\overrightarrow{dE}\right| = \frac{Kq}{x^2}$$

The area of the ring is $\left|\overrightarrow{da}\right| = 2\pi r dr$

$\overrightarrow{da}$ is perpendicular to the plane of the ring and makes an angle θ with $\overrightarrow{dE}$. The flux passing through the small area element of the disc is given by

d ϕ $=\left|\overrightarrow{dE}\right|\left|\overrightarrow{da}\right|cos\theta$

d ϕ $=\frac{Kq}{x^2} \times 2\pi r dr \times \frac{R}{x}$

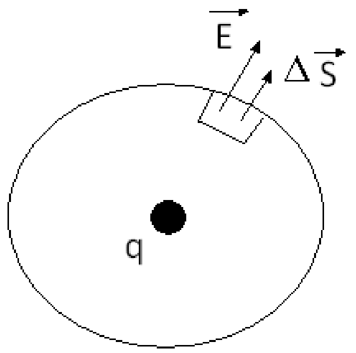
d ϕ $= 2\pi KqR \times \frac{rdr}{x^3}$

d ϕ $= 2\pi KqR \times \frac{rdr}{(R^2+r^2)^{3/2}}$ (as $x^2 = R^2 + r^2$)

Total flux

$$\emptyset = 2\pi kqR\int_0^a \frac{rdr}{(R^2+r^2)^{3/2}} = 2\pi kqR\left[\frac{-1}{\sqrt{R^2-r^2}}\right]_0^a = 2\pi kqR\left[\frac{1}{R} - \frac{1}{\sqrt{R^2-r^2}}\right]$$

Q) Starting form Gauss's law, calculate the electric field due to an isolated point charge q and show that Coulomb's law follows from this result



Solution

For this situation, we choose a spherical Gaussian surface of radius r and centered on the point charge, as shown. The electric field of a positive point charge is radially outward normal to the surface at every point. That is $\vec{E}$ is parallel to $\overrightarrow{dS}$ at each point, and so $\vec{E}.\overrightarrow{dS}$ =EdS.

According to Gauss's law $\phi_E = \oint \vec{E}\,.\,\overrightarrow{dS} = \oint E\,.dS = \frac{q}{\varepsilon_0}$ --(i)

By smmetry, E is constat verywhere on the surface. Therefore

$\oint E\,.dS = E\oint dS = E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

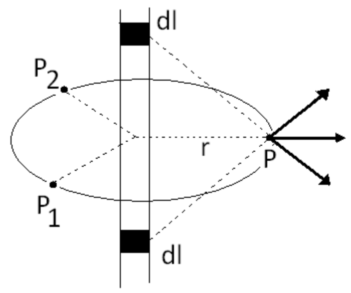
is nothing but Coulomb's law. Hence they are equivalent.

Field due to an infinite line of charge

Consider an infinite line of charge has a linear charge density λ. Using Gauss's law, let us find the electric field at a distance 'r' from the line.

The cylindrical symmetry tells us that the field strength will be the same at all points at a fixed distance r from the line.

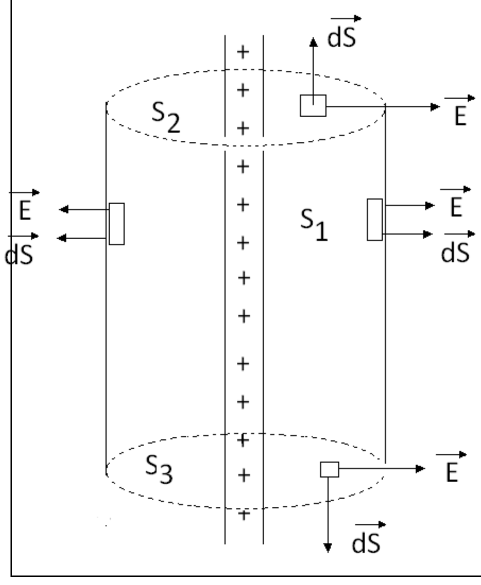
Since the line is infinite and uniform, for every charge element on the other side the component along the line of the fields due to all such element cancels in pairs. Thus the field lines are directed radially outward, perpendicular to the line charge. Also perpendicular distance from line is same magnitude at all points P_1,P_2 will be same



The appropriate choice of Gaussian surface is a cylinder of radius r and length L. On the flat faces S_2 and S_3 , $\vec{E}$ is perpendicular to $\overrightarrow{dS}$, which means no flux cross them.

On curved surface S_1, $\vec{E}$ is parallel to $\overrightarrow{dS}$ so that $\vec{E}.\overrightarrow{dS} = EdS$.

The charge enclosed by the cylinder is Q = λL



Applying Gauss's law to the curved surface, we have

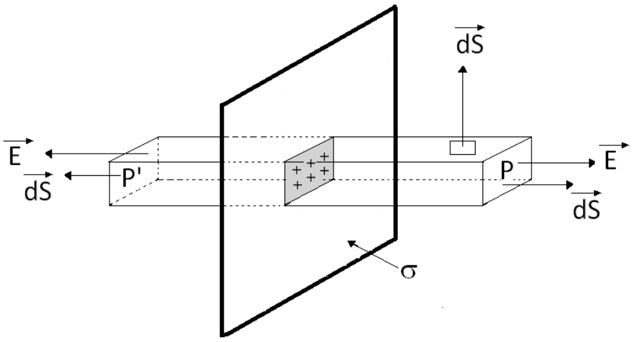
$E\oint dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_0}}$$

Vectorially , $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{r}$ here $\hat{r}$ is the radial unit vector in the plane normal to the wire passing through the point.

Note that although only the charge enclosed by the surface was included above, the electric field E is due to the charges on the entire wire.

Field due to an infinite plane sheet of charge



Let us consider a thin non-conducting plane sheet of charge, infinite in extent and having a surface charge density σ C/m^2 . Let point P be a point at distance r from the sheet, at which the electric intensity is required.

Let us choose a point P' symmetrical with P, on the other side of sheet. Let us now draw a Gaussian surface to be rectangular parallelepiped of cross sectional area A, as shown in figure

By symmetry, the electric field at all points on either side near the sheet will be perpendicular to the sheet, directed outward (if sheet is positively charged). Thus $\vec{E}$ is perpendicular to the plane ends contain point P and P' .Also magnitude of $\vec{E}$ will be same at P and P'. Therefore, the flux passing through the points containing P and P'

$$\emptyset_E = \int \vec{E}.\overrightarrow{dS} + \int \vec{E}.\overrightarrow{dS}$$

$$\emptyset_E = \int EdS + \int EdS$$

$$\emptyset_E = 2EA$$

The flux through remaining surface is zero because $\vec{E}$ is perpendicular to $\overrightarrow{dS}$ and do not contribute to the total flux

The charge enclosed by the Gaussian surface shown by shaded area. q = σA

Applying Gauss's law, we have

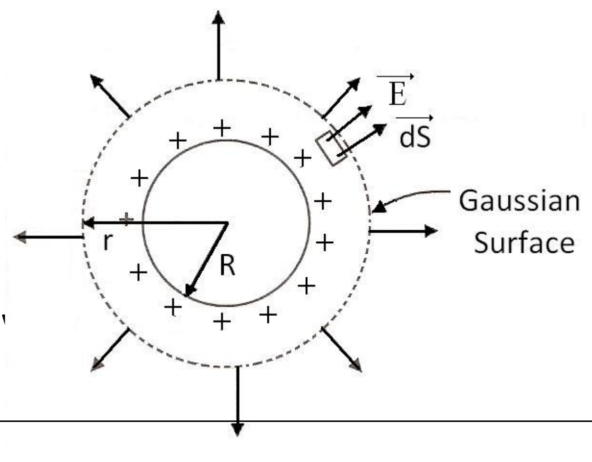
$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma A}{2\varepsilon_0}}$$

Electric field intensity is independent of the distance from an infinite sheet of charges

Electric field due to a uniformly charged spherical shell

Using Gauss's law, we can find the intensity of the electric field due to a uniformly charged spherical shell or a solid conducting sphere at



Case(I): At an external point

In an isolated charged spherical conductor an excess charge on it is distributed uniformly over its surface. Since charge lines is radially

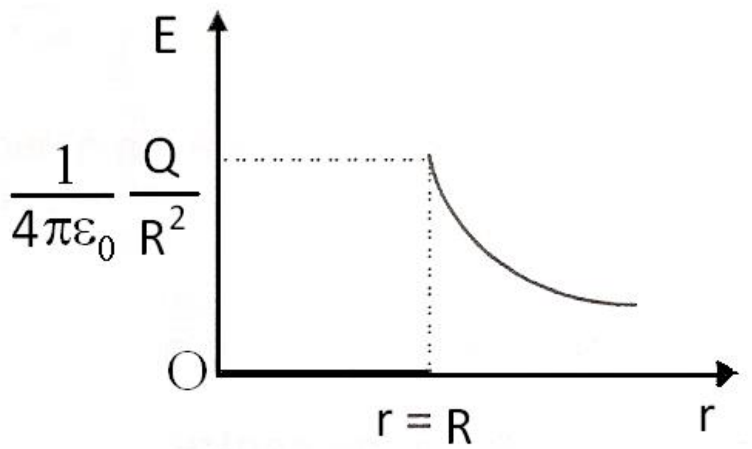
outward. Also, field strength will have the same value at all points on any imaginary spherical surface concentric with the charged conducting sphere or shell. This symmetry leads us to choose the Gaussian surface to be a sphere of radius r > R.
Any arbitrary element of area $\overrightarrow{dS}$ is parallel to the local $\vec{E}$, so $\vec{E}.\overrightarrow{dS}$ = EdS at all points on the surface
According to Gauss's law

$$\oint \vec{E}.\overrightarrow{dS} = \oint EdS = E\oint dS = E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

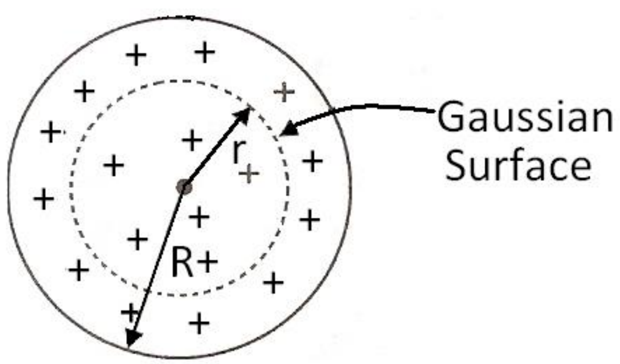
For all points outside the charged conducting sphere or the charge spherical shell, the field is same as that appoint charge at the centre

Case(ii) At an internal point (r <R)
The field still has the same symmetry and so we again pick a spherical Gaussian surface, but now with radius r less than E. Since the enclosed charge is zero, from Gauss' law, we have E = 0
Thus , we conclude that E=0 at all points inside a uniformly charged conducting sphere or the charged spherical shell



Variation of E with the distance from the centre (r)

Electric field due to uniformly charged sphere or sphere of charge



A non conducting uniformly sphere of radius R has a total charge Q uniformly distributed throughout its volume. Using Gauss's law
Positive charge Q is uniformly distributed throughout the volume of sphere of radius R.
Density of charge

Case(i): At an internal point (r <R)
For finding the electric field at a distance (r <R) from the centre, we choose a spherical Gaussian surface of radius r, concentric with the charge distribution. Let charge enclosed is q. From symmetry the magnitude E of the

electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface
So, applying Gauss's law

$$\oint \vec{E}.\overrightarrow{dS} = \oint EdS = E\oint dS = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

Here

$$q = \left(\frac{4}{3}\pi r^3\right)\rho$$

$$q = \left(\frac{4}{3}\pi r^3\right)\frac{3Q}{4\pi R^3} = Q\frac{r^3}{R^3}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}}$$

Or

$$E = \frac{\rho r}{3\varepsilon_0}$$

Case (ii) At an external point (r >R)
To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius r (r>R). This surface encloses the entire charged sphere, So, from Gauss's law, we have

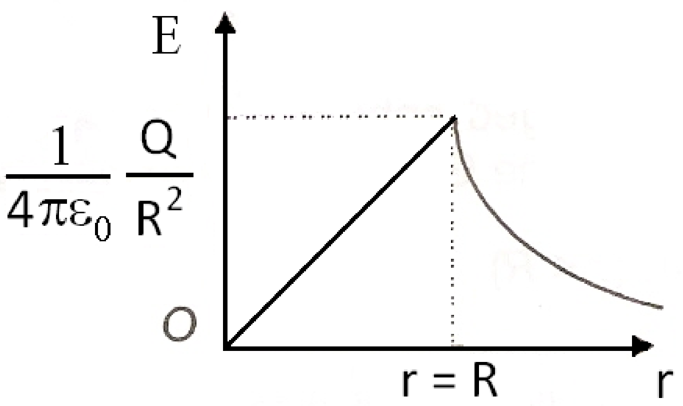
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

Or

$$\boxed{E = \frac{\rho R^3}{3r^2\varepsilon_0}}$$

Variation of E with the distance from centre (r)



QUESTIONS (E)

Q) Give the statement of Gauss' Law

Q) What is the electric field intensity at a point inside a uniformly charged rubber ballon?

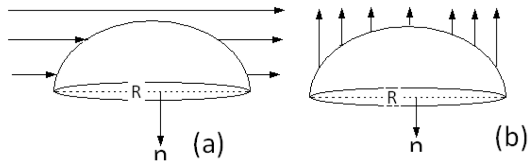
Q) Using Gauss's law, find the intensity of the electric field produced due to uniformly charged infinite plane

Or Prove that electric field intensity is independent of the distance from an infinite sheet of charges

Q) Obtain the expression of electric field due to an infinitely long linear charged wire along the perpendicular distance from the wire.

Q) Using Gauss's Law, find the intensity of the electric field inside and outside the charged sphere having uniform charge density.

Q) A hemispherical body placed in uniform electric field E. What is the flux linked with the curved surface, (i) if field parallel to base fig (a) and (ii)perpendicular to base fig(b)



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