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Magnetic Effect of Current

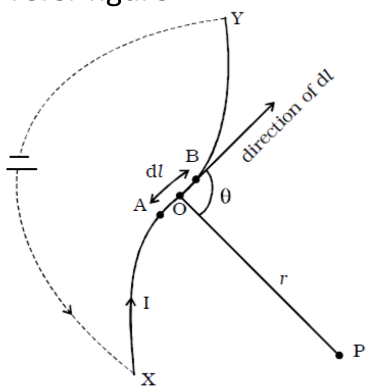
SECTION I

The magnetic field:

Magnetic field is the region around the moving charge in which magnetic force is experienced by the magnetic substances.
Magnetic field is a vector quantity and also known as magnetic induction vector. It is represented by **B**
Lines of magnetic induction may be drawn in the same way as lines of electric field.
The number of lines per unit area crossing a small area perpendicular to the direction of the induction being numerically equal to **B**.
The number of lines of **B** crossing a given area is referred as the magnetic flux linked with that area.
For this reason, **B** is also called magnetic flux density.
The unit of magnetic field is weber/m^2 and also known as tesla (T) in SI system

BIOT-SAVART LAW:

Biot and Savart conducted many experiments to determine the factors on which the magnetic field due to current in a conductor depends. The results of the experiments are summarized as Biot-Savart law. Let us consider a conductor XY carrying a current I refer figure



AB = dl is a small element of the conductor. P is a point at a distance r from the midpoint O of AB. According to Biot and Savart, the magnetic induction dB at P due to the element of length dl is
(i) directly proportional to the current (I)
(ii) directly proportional to the length of the element (dl)
(iii) directly proportional to the sine of the angle between dl and the line joining element dl and the point P (sin θ)
(iv) inversely proportional to the square of the distance of the point from the element ($1/r^2$)

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$dB = K\frac{I\,dl\,\sin\theta}{r^2}$$

K is the constant of proportionality; its value is μ /4π.
Here μ is the permeability of the medium. Value of K for vacuum is 10^{-7} wb/amp m.

$$dB = \frac{\mu}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}$$

$\mu = \mu_r\,\mu_o$ where μ_r is the relative permeability of the medium and μ_0 is the permeability of free space. $\mu_o = 4\pi \times 10–7$ henry/metre. For air $\mu_r = 1$.
So, in air

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}$$

In vector form,

$$dB = \frac{\mu_0}{4\pi}\frac{\overrightarrow{I\,dl} \times \vec{r}}{r^3}$$

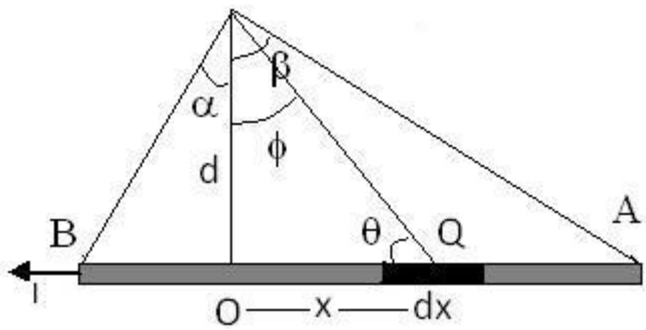
or

$$dB = \frac{\mu_0}{4\pi}\frac{\overrightarrow{I\,dl} \times \hat{r}}{r^2}$$

The direction of dB is perpendicular to the plane containing current element Idl and r (i.e plane of the paper) and acts inwards.
The unit of magnetic induction is tesla (or) weber $m^{-2.}$

Field due to a Straight current carrying wire

(i) When the wire is of finite length
Consider a straight wire segment carrying a current I and there is a point P at which magnetic field to be calculated as shown in figure.
This wire makes an angle of α and β at that point with normal OP. Consider an element of length dx at a distance y from O and distance of this element from point P is r and line joining P and Q makes an angle θ with the direction of current as shown in figure.



Using Biot-Savart Law magnetic field at point due to small current element is given by

$$dB = \frac{\mu_0 I}{4\pi}\left(\frac{dx\sin\theta}{r^2}\right)$$

As every element of the wire contributes to **B** in the same direction, we have

$$B = \frac{\mu_0 I}{4\pi}\int_A^B \left(\frac{dx\sin\theta}{r^2}\right)$$ .....eq(1)

From the triangle OPQ as shown in figure, we have

x = d tanϕ

Or dx = $d\sec^2\phi\, d\phi$

And in same triangle r = d sec ϕ and θ = (90° – ϕ)

Where ϕ is angle between line OP and PQ

Now equation (1) can be written as

$$B = \frac{\mu_0 I}{4\pi}\int_{-\beta}^{\alpha} \left(\frac{d\sec^2\varphi d\varphi\sin(90-\varphi)}{(d\sec\varphi)^2}\right)$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\beta}^{\alpha} \left(\frac{d\sec^2\varphi d\varphi\sin(90-\varphi)}{(d\sec\varphi)^2}\right)$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\beta}^{\alpha} \left(\frac{d\varphi\cos\varphi}{d}\right)$$

$$B = \frac{\mu_0 I}{4\pi d}\int_{-\beta}^{\alpha} (\cos\varphi d\varphi)$$

$$B = \frac{\mu_0 I}{4\pi d}[\sin\varphi]_{-\beta}^{\alpha}$$

$$B = \frac{\mu_0 I}{4\pi d}[\sin\alpha + \sin\beta]$$

Direction of **B**: the direction of magnetic field in determined by the cross product of the vector Idl with vector r

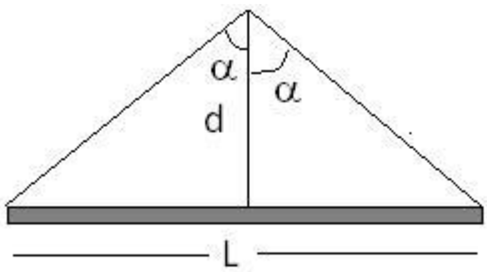
Therefore at point P the direction of the magnetic field due to the whole conductor will be perpendicular to the plane containing wire and point P or perpendicular to plane of paper and going into the plane

Case (I) when point P is on perpendicular bisector

In this case α = β using equation

$$B = \frac{\mu_0 I}{4\pi d}[\sin\alpha + \sin\beta]$$

$$B = \frac{\mu_0 I}{2\pi d}[\sin\alpha]$$



From figure

$$\sin\alpha = \frac{L}{\sqrt{L^2 + 4d^2}}$$

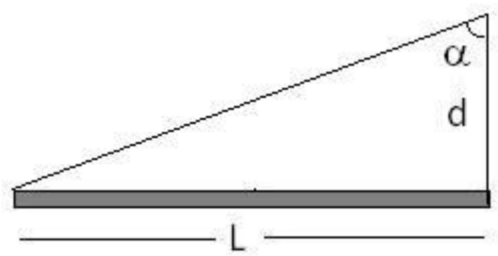
Case (II) when point P is at one end of conductor

In this case α =0 or β = 0

From equation

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\alpha + \sin\beta\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\alpha\right]$$



From figure

$$\sin\alpha = \frac{L}{\sqrt{L^2 + d^2}}$$

Case(III) When wire is of infinite length

In this case α = β = 90^O

From equation

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\alpha + \sin\beta\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin 90 + \sin 90\right]$$

$$B = \frac{\mu_0 I}{2\pi d}$$

Case(IV) When the point P lies along the length of wire (but not on it)

If the point is along the length of wire (but not on it) , then as vector dl and vector r will either be parallel or antiparallel i.e θ=0 or π,

From equation

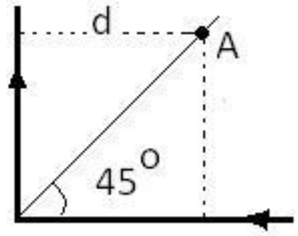
$$dB = \frac{\mu}{4\pi}\frac{\mathrm{I\ dl\ sin\ }\theta}{r^2}$$

$$dB = \frac{\mu}{4\pi}\frac{\mathrm{I\ dl\ sin\ 0}}{r^2}$$

$$dB = 0$$

Solved Problem

Q) A along straight conductor is bent at an angle of 90^O as shown in figure. Calculate the magnetic field induction at A



Solution:
For each portion α = 45 and β =90 .
From formula for magnetic field at a point

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin\alpha + \sin\beta\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin 45 + \sin 90\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\frac{1}{\sqrt{2}} + 1\right]$$

$$B = \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right]$$

Each horizontal and vertical wires will produce same magnetic field at A and there directions are also same thus total field at A is

$$B = 2\times\frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right]$$

$$B = \frac{\mu_0 I}{2\pi d}\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right]$$

Q) A long straight wire carrying current produces a magnetic induction of 4×10^{-6}T at a point, 15 cm from the wire. Calculate the current through the wire.

Solution:
B =4×10^{-6}T , d = 15 cm = 0.15m
From formula

$$B = \frac{\mu_0 I}{2\pi d}$$

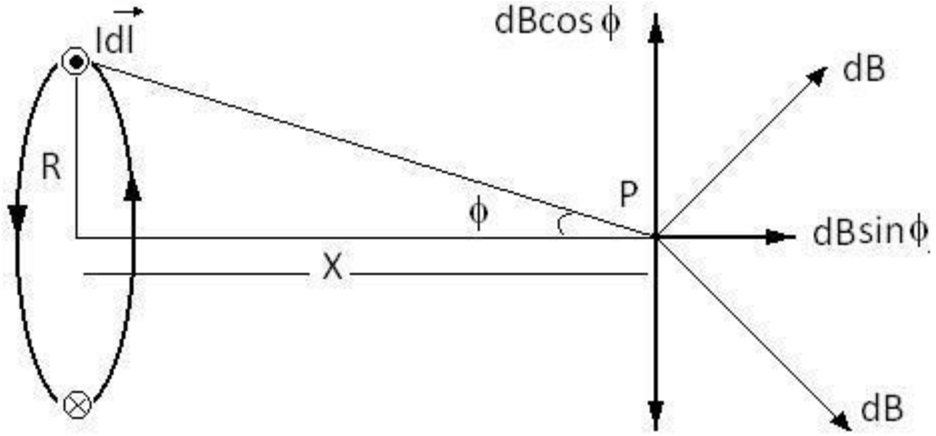
$$I = \frac{B(2\pi d)}{\mu_0}$$

$$I = \frac{4\times10^{-6}\times2\pi\times0.15}{4\pi\times10^{-7}}$$

$$I = 3A$$

Magnetic field at an axial point of a circular coil

Consider a circular loop of radius R and carrying a steady current I. We have to find out magnetic field at the axial point P, which is at distance x from the centre of the loop
X-axis is taken as along the axis of the ring.
Let the position vector of point P with respect to an element **dl** be **r** . The magnetic field dB at point due to current element I**dl** is in a direction perpendicular to the plane formed by **dl** and **r.**



Magnetic field at point P due to current element **Idl** is given by

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\vec{r}}{r^3}$$

Since angle between Idl and r is 90° we get

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{idl}{r^2}\hat{n}$$

Direction of magnetic field is perpendicular to plane containing dl and r as shown in figure
If ϕ is the angle between r and X, then from geometry of figure, component of **dB** along Y-axis will be dBcosϕ and dBsinϕ will be along X axis.
For all point on the circular coil there exists a diametrically opposite point such that magnetic field produced at point P cancels Y-component of first one thus resultant magnetic field at P is summation of X-component at P

$$B = \int dB \sin\varphi$$

$$B = \frac{\mu_0}{4\pi}\int \frac{Idl}{r^2}\sin\varphi$$

$$\sin\varphi = \frac{R}{r}$$

$$\sin\varphi = \frac{R}{\sqrt{R^2 + x^2}}$$

$$B = \frac{\mu_0}{4\pi}\int \frac{Idl}{r^2}\frac{R}{\sqrt{R^2 + x^2}}$$

$$B = \frac{\mu_0}{4\pi}\int \frac{Idl}{R^2 + x^2}\frac{R}{\sqrt{R^2 + x^2}}$$

$$B = \frac{\mu_0}{4\pi}\int \frac{IRdl}{\left(R^2 + x^2\right)^{3/2}}$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{\left(R^2 + x^2\right)^{3/2}}\int_0^{2\pi R} dl$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{\left(R^2 + x^2\right)^{3/2}}\left(2\pi R\right)$$

$$B = \frac{\mu_0}{2}\frac{IR^2}{\left(R^2 + x^2\right)^{3/2}}$$

Case (I) If the coil has N turns then

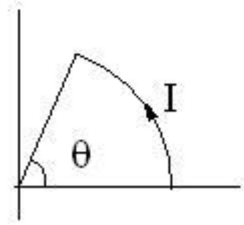
$$B = \frac{\mu_0}{2}\frac{NIR^2}{\left(R^2 + x^2\right)^{3/2}}$$

Case (II) Field at the centre of ring
In above equation x = 0

$$B = \frac{\mu_0}{2}\frac{NIR^2}{R^3}$$

$$B = \frac{\mu_0 NI}{2R}$$

Case (III) Magnetic field at the centre of a current arc



Form the equation for magnetic field at centre of coil. N is number of turns

2π = 1 turn

∴ θ = θ/2π turn s replacing value of N we get

$$B = \frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$

If l is the length of arc then l = θR above equation becomes

$$B = \frac{\mu_0 I}{2R}\frac{1}{2\pi}\frac{l}{R}$$

$$B = \frac{\mu_0 Il}{4\pi R^2}$$

Direction of magnetic field **B** for circular loop

Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained using the right-hand thumb rule. If the fingers curled along the current then stretched thumb shows direction of magnetic field.

Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current

Solved Problem

Q) A circular coil of 200 turns and of radius 20 cm carries a current of 5A. Calculate the magnetic induction at a point along its axis, at a distance three times the radius of the coil from its centre

Solution:

N=200, R = 0.2 m, I =5A , x = 3R

From the formula

$$B = \frac{\mu_0}{2}\frac{NIR^2}{\left(R^2 + x^2\right)^{3/2}}$$

$$B = \frac{\mu_0}{2}\frac{NIR^2}{\left(R^2 + 9R^2\right)^{3/2}}$$

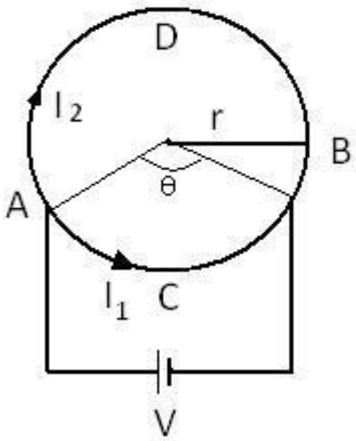
$$B = \frac{\mu_0}{2}\frac{NI}{10^{3/2} R}$$

$$B = \frac{\mu_0 NI}{20\sqrt{10}R}$$

$$B = \frac{4\pi\times 10^{-7}\times 200\times 5}{20\sqrt{10}\times 0.2}$$

$$B = 9.9\times 10^{-5} T$$

Q) A circular loop is prepared from a wire of uniform cross section. A battery is connected between any two points on its circumference. Show that the magnetic induction at the centre of the loop is zero



Solution:

Magnetic field at centre due to arc is given by

$$B_1 = \frac{\mu_0 I_1}{2R}\frac{\theta}{2\pi}$$

and

$$B_2 = \frac{\mu_0 I_2}{2R}\frac{(2\pi - \theta)}{2\pi}$$

Both magnetic fields B_1 and B_2 are in opposite directions

Let R_1 be the resistance of arc ACB and R_2 be the resistance of arc ADB since potential across both the resistance is same thus

$I_1R_1 = I_2R_2$ eq(1)

We also know that resistance ∝ length of wire

And lenth of wire ACB = rθ

Length of arc ADB = r(2π –θ)

Thus if ρ is resistance per unit length then

$R_1 = \rho r\theta$ and $R_2 = \rho r(2\pi - \theta)$

Thus equation (1) becomes

$I_1 \rho r\theta = I_2 \rho r(2\pi - \theta)$

$I_1 \theta = I_2 (2\pi - \theta)$ eq(2)

Now total magnetic field at centre

$$B = \frac{\mu_0 I_1}{2R}\frac{\theta}{2\pi} - \frac{\mu_0 I_2}{2R}\frac{(2\pi - \theta)}{2\pi}$$

from eq(3)

$$B = \frac{\mu_0 I_1}{2R}\frac{\theta}{2\pi} - \frac{\mu_0 I_1}{2R}\frac{\theta}{2\pi} = 0$$

Q) A charge Q is uniformly spread over a disc of radius R made from non conducting material. This disc is rotated about its geometrical axis with frequency f. Find the magnetic field produced at the centre of the disc.

Solution:

Suppose disc with radius R is divided into the concentric rings with various radii,
Consider one such ring with radius r and thickness dr.
Total charge on disc = Q , charge per unit area $\rho = Q/\pi R^2$
∴ The charge on the ring with radius r = (area of the ring) (charge per unit area)
$q = (2\pi r dr)(Q/\pi R^2)$
If the ring is rotating with frequency f, then current produced I

$$I = \frac{Q}{\pi R^2}(2\pi r dr) f$$

This ring can be considered as circular loops carrying current I
Magnetic field at the centre due to this current will be

$$dB = \frac{\mu_0}{2r} I$$

$$dB = \frac{\mu_0}{2r}\frac{Q}{\pi R^2}(2\pi r dr) f$$

$$dB = \frac{\mu_0 Q f}{R^2}(dr)$$

∴ Magnetic field B produced at the centre due to the whole disc

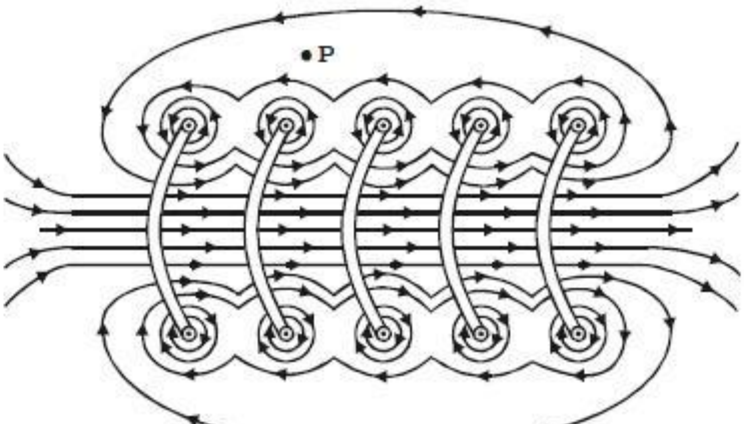
$$B = \int dB = \int_0^R \frac{\mu_0 Q f}{R^2} dr$$

$$B = \frac{\mu_0 Q f}{R}$$

Solenoid:

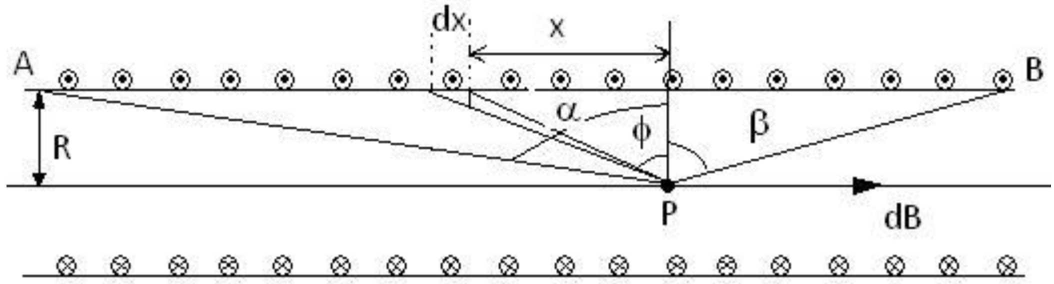
When two identical rings carrying current in same direction are placed closed to each other co-axially. It is obvious that the magnetic field produced due to the rings is in same direction on the common axis. Moreover the lines close to the axis are almost parallel to the axis and in the same direction.

Thus if a number of such rings are kept very closed to each other and current is passed in the same direction.



The magnetic fields associated with each single turn are almost concentric circles and hence tend to cancel between the turns. At the interior midpoint, the field is strong and along the axis (i.e) the field is parallel to the axis. For a point such as P, the field due to the upper part of the solenoid turns tends to cancel the field due to the lower part of the turns, acting in opposite directions. Hence the field outside the circular coil is very less
Solenoid is a device in which this situation is realized.
A helical coil consisting of closely wound turns of insulated conducting wire is called solenoid.
When length of a solenoid is very large compared to its radius, the solenoid is called long solenoid. For long solenoid magnetic field outside is practically zero.

Magnetic field at a point on the axis of the a SHORT solenoid



Consider a solenoid of length L and radius R containing N closely spaced turns and carrying steady current I. let number of turns per unit length be n
The field at point P on the axis of a solenoid can be obtained by superposition of fields due to large number of turns all having their centre on the axis of the solenoid as shown in figure
Consider a coil of width dx at a distance x from the point P on the axis as shown in figure
The field at P due to ndx turns is

$$dB = \frac{\mu_0}{2}\frac{(ndx)IR^2}{(R^2+x^2)^{3/2}}$$

From figure x = R tanϕ
dx = $R\sec^2\phi\, d\phi$
On substituting values in above equation we get

$$dB = \frac{\mu_0}{2}\frac{\left(nR\sec^2\varphi d\varphi\right)IR^2}{\left(R^2+R^2\tan^2\varphi\right)^{3/2}}$$

$$dB = \frac{\mu_0}{2}\frac{nI\sec^2\varphi}{\sec^3\varphi}d\varphi$$

$$dB = \frac{\mu_0}{2}nI\cos\varphi d\varphi$$

$$B = \int_{-\alpha}^{\beta}\frac{\mu_0}{2}nI\cos\varphi d\varphi$$

$$B = \frac{\mu_0}{2}nI\int_{-\alpha}^{\beta}\cos\varphi d\varphi$$

$$B = \frac{\mu_0}{2}nI\left[\sin\alpha+\sin\beta\right]$$

Case (I) If the solenoid is of infinite length and the point is well inside the solenoid
In this case α = β = π/2 then B is

$$B = \frac{\mu_0}{2}nI\left[\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\right]$$

$$B = \mu_0 nI$$

Case(II) If the solenoid is of INFINITE length and the point is near one end
In this case α = 0 and β = π/2

$$B = \frac{\mu_0}{2}nI\left[\sin 0+\sin\frac{\pi}{2}\right]$$

$$B = \frac{\mu_0}{2}nI$$

Case (III) If the solenoid is of FINITE length and the point is on the perpendicular bisector of its axis
In this case α = β

$$B = \frac{\mu_0}{2}nI\left[\sin\alpha+\sin\alpha\right]$$

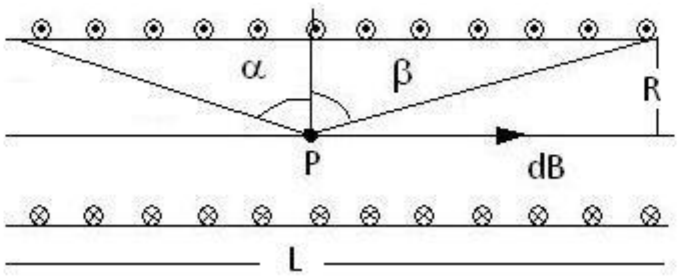
$$B = \mu_0 nI\sin\alpha$$

$$\sin\alpha = \frac{L}{\sqrt{L^2+4R^2}}$$

Solved problem

Q) A solenoid of length 0.4m and diameter 0.6m consists of a single layer of 1000turns of fine wire carrying a current of 5 ×10^{-3}ampere. Calculate the magnetic field on the axis of the middle and at the end of the solenoid

<u>Solution:</u>



In case of a finite solenoid, the field at the point on the axis is given by

$$B = \frac{\mu_0}{2} nI\left[\sin\alpha + \sin\beta\right]$$

$$n = \frac{N}{L} = \frac{1000}{0.4} = 2.5\times 10^3$$

$$B = 2.5\pi\times 10^{-6}\left[\sin\alpha + \sin\beta\right]$$

a) Middle point α = β thus
   B = 2.5π×10^{-6}(2sinα) and

$$\sin\alpha = \frac{L}{\sqrt{L^2 + 4R^2}}$$

$$\sin\alpha = \frac{L}{\sqrt{L^2 + d^2}}$$

$$\sin\alpha = \frac{0.4}{\sqrt{(0.4)^2 + (0.6)^2}}$$

$$\sin\alpha = \frac{4}{7.2}$$

   B = 2.5π×10^{-6}×2×(4/7.2) = 8.72×10^{-6} T

b) When the points is at the end on axis

$$\sin\beta = \frac{L}{\sqrt{L^2 + R^2}}$$

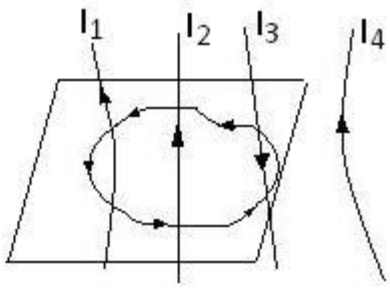
$$\sin\beta = \frac{0.4}{\sqrt{(0.4)^2 + (0.3)^2}}$$

$$\sin\beta = \frac{4}{5}$$

   B = 2.5π×10^{-6}×2×(4/5) = 6.28×10^{-6} T

Ampere's Law

Consider electric currents I_1,I_2, I_3, I_4 as shown in figure. All these current produce magnetic field in the region around electric current.
A plane which is not necessarily horizontal is shown in figure. An arbitrary closed curve is also shown in figure



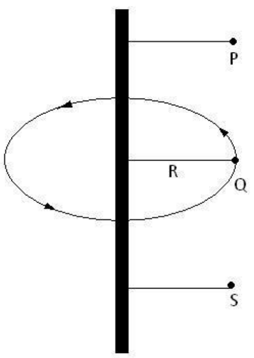
Sign convention for current:
Arrange right hand screw perpendicular to plane containing closed loop and rotate in the direction of vector element taken for line integration. Electric current in the direction of advancement of screw is considered positive and current in opposite direction are considered negative.
Now from sign convention I_1 and I_2 are positive while I_3 is negative.
Hence algebraic sum $\sum I = I_1 + I_2 - I_3$
Here we don't worry about current not enclosed by the loop
The statement of the Ampere's Law is as under:
The line integral of magnetic induction over a closed loop in a magnetic field is equal to the product of algebraic sum of electric current enclosed by loop and the magnetic permeability"
The law can be represented mathematically as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum I$$

The magnetic induction in the above equation is due to all current. Whereas algebraic sum of current on right hand side is only of those currents which are enclosed by the loop
This law is true for steady current.

Application of Ampere's Law

(A) To find Magnetic field Due to a very long straight conductor carrying electric current



Let I be the current flowing through a very long conductor. Now consider points like P, Q, S located at same perpendicular distance R from wire. Since the two ends of wire at infinity and due to symmetry of wire magnetic field at pints P, Q and S is same.
Thus magnetic field at all point on the circumference of circle of radius R must be same. Or B is constant
Consider a small segment of length dl along the circumference.
Now by applying Ampere's Law we get

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \sum I$$

$$\oint B\,dl\cos\theta = \mu_0 I$$

$\vec{B}$ *and dl are in same direction*

$$\oint B\,dl = \mu_0 I$$

$$B\oint dl = \mu_0 I$$

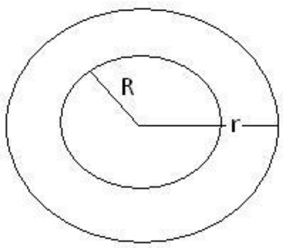
$$B(2\pi R) = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{R}$$

Thus Outside the conductor B ∝ (1/R)

Magnetic field inside the conductor

Consider a top view of conductor of radius r . We want to find magnetic field at a distance R < r. Consider a loop of radius R as shown in figure



Let conductor carries a current I thus current through conductor of radius R is

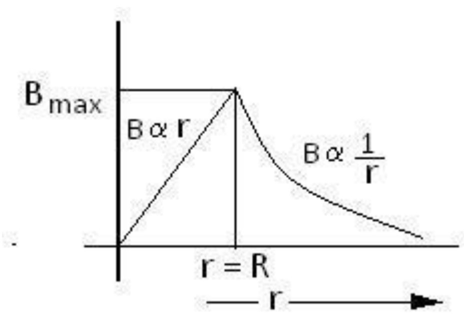
$$i = \left(\frac{I}{\pi r^2}\right)\pi R^2 = I\frac{R^2}{r^2}$$

From Ampere's Law

$$B(2\pi R) = \mu_0 i$$

$$B(2\pi R) = \mu_0 I\frac{R^2}{r^2}$$

$$B = \left(\frac{\mu_0 I}{2\pi r^2}\right)R$$

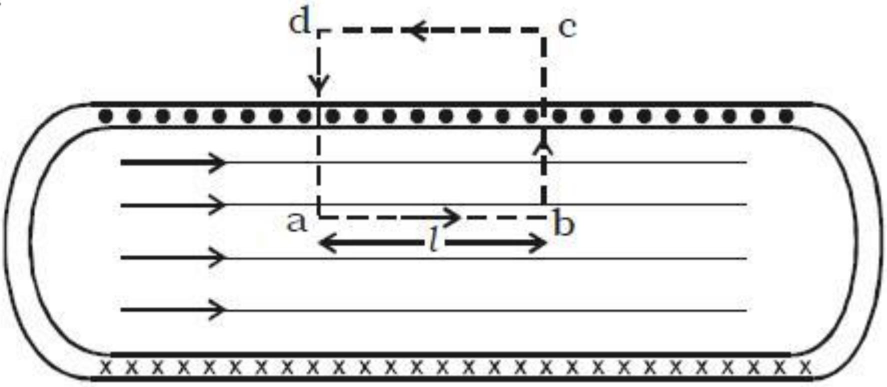


For inside the conductor B∝ R

Hence for magnetic field

(i) Outside the conductor B ∝ (1/R)
(ii) Inside the conductor B ∝ R
(iii) On the conductor Maximum
(iv) At end points outside conductor = 0

(B) Magnetic field inside a LONG solenoid using Ampere's Circuital Law



A solenoid is a wire wound closely in the form of a helix, such that adjacent turns are electrically insulated

The magnetic field inside a very tightly wound long solenoid is uniform everywhere along the axis of the solenoid and is zero outside it.

To calculate the magnetic field at point 'a', let us draw rectangle abcd as shown in figure.

The line ab is parallel to the solenoid axis and hence parallel to magnetic field **B** inside the solenoid thus **B.dl** = B(dl)

Line da and bc are perpendicular thus **B.dl** = 0

Line cd is outside the solenoid here B = 0 thus **B.dl** = 0

If i is the current and n is the number of turns per unit length then current enclosed by the loop = ni l

From Ampere's Law

$$\int_a^b B dl = \mu_0 nil$$

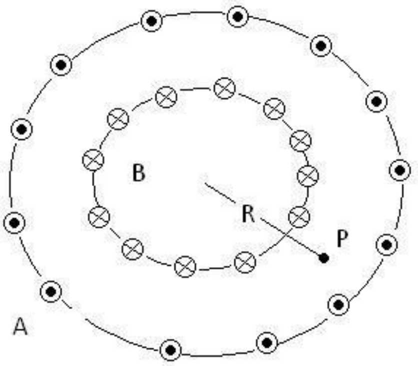
$$Bl = \mu_0 nil$$

$$B = \mu_0 ni$$

Toroid:

If a solenoid is bent in the form of a circle and its two ends are joined with each other the device is called a toroid.

The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. It can be viewed as a solenoid which has been bent into a circular shape to close itself. It is shown in figure carrying a current i



Magnetic field at point A and B is zero as points are outside the toroid
Magnetic field at point P inside the toroid which is at distance R from its centre as shown in figure. Clearly magnetic field at point on the circle of radius R is constant. And directing towards the tangent to the circle. Therefore

$$\oint \vec{B}\cdot\vec{dl} = B(2\pi R)$$

If total number of turns is N and current passing is I, the total current passing through said loop must be NI
From Ampere's Law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 NI$$

$$B(2\pi R) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi R}$$

$$B = \mu_0 nI$$

Here n = N/2πR the number of turns per unit length of toroid
This magnetic field is uniform at each point inside toroid
In an ideal toroid, the turns are completely circular. In such toroid magnetic field inside the toroid is uniform and outside is zero.
But toroid used in practice turns are helical and hence small magnetic field is produced outside the toroid
Toroid is used for nuclear fusion devise for confinement of plasma.

SECTION II

Force on a charged particle in a magnetic field

When a charged 'q' moving in a magnetic field **B** with velocity **v** then force experienced by the charge is given by

$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

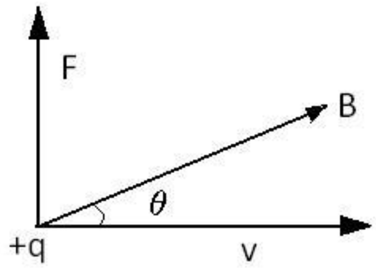
The magnitude is given by

$F = qvB\sin\theta$ Here θ is angle between direction of **B** and direction of velocity **v**

Direction of force is perpendicular to both **B** and **v**

The right hand thumb rule:

For determining the direction of cross product **v** × **B**, you point the four fingers of your right hand along the direction of v, and palm in the direction of magnetic field B the curl the fingers. The thumb is then points in the direction of **v** × **B**



If q is negative then direction of F will be opposite to direction of **v** × **B**

Important points:

(1) The magnetic force will be maximum when $\sin\theta = 1 \Rightarrow \theta = 90^o$
Change is moving perpendicular to magnetic field $F_{max} = qvB$
In this situation F, v, B are mutually perpendicular to each other.

(2) The magnetic force will be minimum when $\sin\theta = 0 \Rightarrow \theta = 0$ or 180^o
It means charge is moving parallel to magnetic field $F_{min} = 0$

(3) Magnetic force is zero when charge is stationary

(4) In case of motion of charged particle in a magnetic field, as the force is always perpendicular to direction of charge work done is zero. Or magnetic force cannot change kinetic energy of charge and speed remains constant

Difference between Electric and Magnetic field

(1) Magnetic force is always perpendicular to the field while electric force is collinear with the field

(2) Magnetic force is velocity dependent i.e. acts only when charged particle is in motion while electric force is independent of the state of rest or motion of the charge.

(3) Magnetic force does not work when the charged particle is displaced while electric force does work in displacing the charged particle.

(4) Magnetic force is always non-central while the electric force may or may not be.

Non-central force: A force between two particles that is not directed along the line connecting them.

Motion of a charged particle in a uniform magnetic field

(A) When the charged particle is given velocity perpendicular to the field:
Let a particle of charge q and mass m is moving with a velocity 'v' and enters at right angles to uniform magnetic field N as shown in figure
The force on the particle is qvB and this force will always act in a direction perpendicular to v. Hence, the particle will always act in a direction perpendicular to v. Hence the particle will move on a circular path. If the radius of the path is r then

$$\frac{mv^2}{r} = Bqv$$

$$r = \frac{mv}{qB}$$

Thus radius of the path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of magnetic field
Time period:
The time period is the time taken by the charged particle to complete on rotation of the circular path which is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The time period is independent of the speed
Frequency:
The frequency is the number of revolution of charged particle in one second which is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

SOLVED NUMERICAL

Q) Two particles of mass M and m and having equal electric charge are accelerated through equal potential difference and then move inside a uniform magnetic field, normal to it. If the radii of the circular paths are R and r respectively find the ratio of their masses
<u>Solution:</u>
Since charge is same on both the particles and are accelerated through equal potential both particles will have same kinetic energy. Let p_1 be the momentum of particle of mass M and p_2 be the momentum of particle of mass 'm' thus

$$\frac{p_1^2}{2M} = \frac{p_2^2}{2m}$$

$$\frac{p_1^2}{p_2^2} = \frac{M}{m}$$

From the equation for radius r ∝ momentum
Thus

$$\frac{M}{m} = \left(\frac{R}{r}\right)^2$$

(B) When a charged particle is moving at an angle to the field

In this case the charged particle having charge q and mass m is moving with velocity v and it enters the magnetic field B at angle θ as shown in figure. Velocity can be resolved in two component one along magnetic field and the other perpendicular to it. Let these components are $V_{||}$ and $V_{\perp}$

$V_{||} = V\cos\theta$ and $V_{\perp} = V\sin\theta$

The parallel component $V_{||}$ of velocity remains unchanged as it is parallel to B.

Due to perpendicular component $V_{\perp}$ the particle will move on a circular path.

So resultant path will be combination of straight line motion and circular motion, which will be helical path

The radius of path:

$$r = \frac{mv\sin\theta}{qB}$$

Time period:

$$T = \frac{2\pi r}{v_{\perp}}$$

$$T = \frac{2\pi mv\sin\theta}{v\sin\theta qB}$$

$$T = \frac{2\pi m}{qB}$$

Frequency (f)

$$f = \frac{qB}{2\pi m}$$

Pitch :

Pitch of helix described by charged particle is defined as the distance moved by the centre of circular path in the time in which particle completes one revolution

Pitch = $V_{||}$ (time period)

$$pitch = v\cos\theta\frac{2\pi m}{Bq}$$

$$pitch = \frac{2\pi mv\cos\theta}{Bq}$$

Motion of charged particle in combined electric and magnetic field

When the moving charged particle is subjected simultaneously to both electric field and magnetic field B, the moving charged particle will experience electric force $\mathbf{F_e} = q\mathbf{E}$ and magnetic force $\mathbf{F_m} = q(\mathbf{v} \times \mathbf{B})$ **,** so the net force on it will be

$\mathbf{F} = q[E + (\mathbf{v} \times \mathbf{B})]$ which is called Lorentz force equation**:**

Case (I) when v, B, and E all three are collinear

In this situation as the particle is moving parallel or antiparallel to the field, the magnetic force on it will be zero and only electric force will act so

$\vec{a} = \frac{q\vec{E}}{m}$

Hence particle will flow straight path with changing speed and hence kinetic energy, momentum will also change

Case(II) **v, E** and **B** are mutually perpendicular

v, E and **B** are mutually perpendicular.

In case situation of E and B are such that

$F = \mathbf{F_e} + \mathbf{F_m} = 0$

Then a = 0, particle will move in its original without change in velocity in this situation

qE = qvB

or v = E/B

This principle is used in velocity selector to get a charged beam having a specific velocity

SOLVED PROBLEM

Q) A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with velocity 1.28×10^{6} m/s in +x direction enters a region in which a uniform electric field E and a uniform magnetic field b are present such that $E_x = E_y = 0$; $E_z = 102.4$ kV/m and $B_x = B_z = 0$, $B_z = 8\times10^{-2}$ wb/m^2. The particle enters in a region at the origin at time t = 0. Find the location (x, y, z) of the particle at $t = 5\times10^{-6}$s

Solution

From Lorentz equation

$\mathbf{F} = q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$

$F=q[102.4\times10^{3}\,\mathbf{i} + (1.28\times10^{6}\,\mathbf{i}\times8\times10^{-2}\,\mathbf{k})]$

$F=q[102.4\times10^{3}\,\mathbf{i} + (-102.4\times10^{3}\,\mathbf{i})]$

$\mathbf{F} = 0$

Hence, the particle will move along + x axis with constant velocity 1.28×10^{6} m/s

X=vt = 6.40 m

Location is (6.4, 0, 0)

Cyclotron

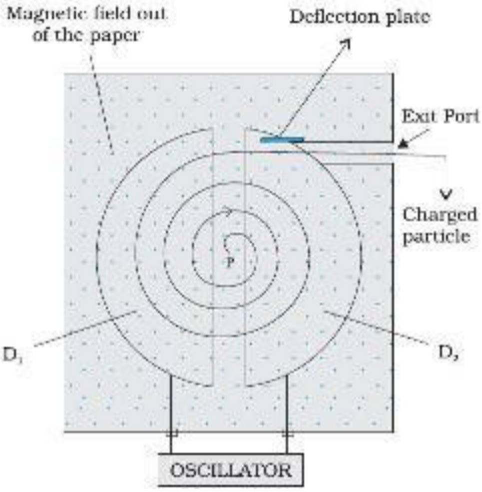
Cyclotron is a device used to accelerate charged particles to high energies. It was devised by Lawrence.

Principle

Cyclotron works on the principle that a charged particle moving normal to a magnetic field experiences magnetic Lorentz force due to which the particle moves in a circular path.

Construction

It consists of a hollow metal cylinder divided into two sections D_1 and D_2 called Dees, enclosed in an evacuated chamber. The Dees are kept separated and a source of ions is placed at the centre in the gap between the Dees. They are placed between the pole pieces of a strong electromagnet. The magnetic field acts perpendicular to the plane of the Dees. The Dees are connected to a high frequency oscillator. The whole assembly is evacuated to minimize collisions between the ions and the air molecules. A high frequency alternating voltage is applied to the Dees. In the sketch shown in Fig. positive ions or positively charged particles (e.g., protons) are released at the centre P.



Working

When a positive ion of charge *q* and mass *m* is emitted from the source, it is accelerated towards the Dee having a negative potential at that instant of time. Due to the normal magnetic field, the ion experiences magnetic Lorentz force and moves in a circular path. By the time the ion arrives at the gap between the Dees, the polarity of the Dees gets reversed. Hence the particle is once again accelerated and moves into the other Dee with a greater velocity along a circle of greater radius. Thus the particle moves in a spiral path of increasing radius and when it comes near the edge, it is taken out with the help of a deflector plate (D.P). The particle with high energy is now allowed to hit the target. When the particle moves along a circle of radius *r* with a velocity *v*, the magnetic Lorentz force provides the necessary centripetal force. They move in a semi-circular path in one of the dees and arrive in the gap between the dees in a time interval *T*/2; where *T,* the period of revolution

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

The time taken to describe a circle

$$r = \frac{mv}{Bq}$$

$$r = \frac{m\omega r}{Bq}$$

$$\omega = \frac{Bq}{m}$$

$$But\ \omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \frac{Bq}{m}$$

$$T = \frac{2\pi m}{Bq}$$

$$f = \frac{Bq}{2\pi m}$$

It is clear from equation that the time taken by the ion to describe a circle is independent of (i) the radius (r) of the path and (ii) the velocity (*v*) of the particle
So, in a uniform magnetic field, the ion traverses all the circles in exactly the same time.
If the high frequency oscillator is adjusted to produce oscillations of frequency as given in equation resonance occurs. Cyclotron is used to accelerate protons, deutrons and α - particles.
Limitations
(i) Maintaining a uniform magnetic field over a large area of the Dees is difficult.
(ii) At high velocities, relativistic variation of mass of the particle upsets the resonance condition.
(iii) At high frequencies, relativistic variation of mass of the electron is appreciable and hence electrons cannot be accelerated by cyclotron.

Solved numerical

Q) A particle having 2C charge passes through magnetic field of 4 k T and some uniform electric field with velocity 25j. IF Lorentz force acting on it is 400 I N. find the electric field in this region
Solution
Lorentz force
F =q[**E** +(**v** ×**B**)]
400 **i** = 2 [E + 25(4) (**j** ×**k**)]
400 **i** = 2E +200**i**
E = 100 **i** V/m

Force on current carrying conductor in magnetic field

Let L be the length of the straight conductor carrying current I and placed perpendicular to a uniform magnetic induction B

A current in a conductor is due to flow of charge

If v_d is drift velocity of charge A is cross section of conductor n is density of charge per unit volume then from equation

$I = nqv_dA$

Now number of charges in conductor of length L is N = nAL

The force on N charges $F = BNqv_d$

Total force $F = NnALqv_d$

From equation for current

F = BIL

In vector form **F** = I (**L** × **B**)

Where L is a vector in the direction of the current, magnitude of **L** is L for the segment of wire in a uniform magnetic field

F = ILBsinθ here θ is the angle between vector IL and **B**

Magnitude of the force

The magnitude of the force is F = BIL sin θ

(i) If the conductor is placed along the direction of the magnetic field, $\theta = 0^o$, Therefore force F = 0.

(ii) If the conductor is placed perpendicular to the magnetic field,

$\theta = 90^o$, F = BI*l.* Therefore the conductor experiences maximum force.

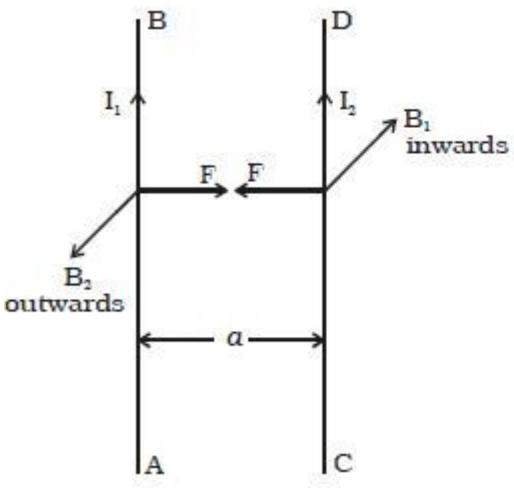
(iii) Force on a closed loop of an arbitrarily shaped conductor is zero

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming's Left Hand Rule.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

Force between two long straight parallel current carrying conductors

AB and CD are two straight very long parallel conductors placed in air at a distance *a*. They carry currents I_1 and I_2 respectively.



The magnetic induction due to current I_1 in AB at a distance *a* is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad \text{...eq(1)}$$

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I_2 is situated in this magnetic field. Hence, force on a
segment of length L of CD due to magnetic field B_1 is

$F = B_1 I_2 L$

substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I_2 flowing in CD at a distance *a* is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad \text{.....eq(3)}$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I1, is situated in this field. Hence force on a segment of length *l* of AB due to magnetic field B2 is

$$F = B_2 I_1 L$$

substituting equation (3)

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length L is

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

Force per unit length of the conductor is

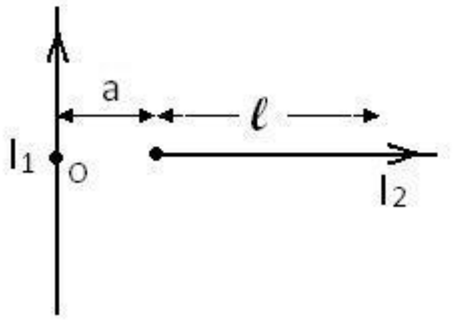
$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1$ Amp , a = 1 m Then $F/L = 2 \times 10^{-7}$

The above conditions lead the following definition of ampere. Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one metre apart, experience a force of 2 × 10^{-7} newton per unit length of the conductor.

Solved Numerical

Q) As shown in figure very long conductor wire carrying current I_1 is arranged in y direction another conducting wire of length l carrying current I_2 is placed along X-axis at a distance a from this wire. Find the torque acting on this wire with respect to point O



Solution:

We can consider wire of current I_2 is in the magnetic field produced current I_1

The force acting on a current element I_2dx located at a distance x from O is,

dF = I_2dx **I** × **B**

Here B is

$$B = \frac{\mu_0 I_1}{2\pi x}\left(-\hat{k}\right)$$

Thus

$$d\vec{F} = I_2 dx\hat{i} \times \frac{\mu_0 I_1}{2\pi x}\left(-\hat{k}\right)$$

$$d\vec{F} = \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j}$$

The torque acting on this element with respect to O is

dτ = x**i** × d**F**

$$d\vec{\tau} = xi \times \frac{\mu_0 I_1 I_2 dx}{2\pi x} j$$

$$d\vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi} dx\hat{k}$$

Total torque acting on this coil can be obtained by taking integration of this equation between x = 0 to x = a + l

$$\vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\int_a^{a+l} dx\hat{k}$$

$$\vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\left[x\right]_a^{a+l}$$

$$\vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\left[a + l - a\right]\hat{k}$$

$$\vec{\tau} = \frac{\mu_0 I_1 I_2 l}{2\pi}\hat{k}$$

Q) A straight wire of length 30cm and mass 60mg lies in a direction 30^o east of north. The earth's magnetic field at this is in horizontal and has a magnitude of 0.8 $\times 10^{-4}$T. What current must be passes through the wire so that it may float in air? [g = 10 m/s^2]

Solution:

$$F = BIL\sin\theta$$

This force will act upward should be equal to downward force of gravitation = mg thus

$$mg = BIL\sin\theta$$

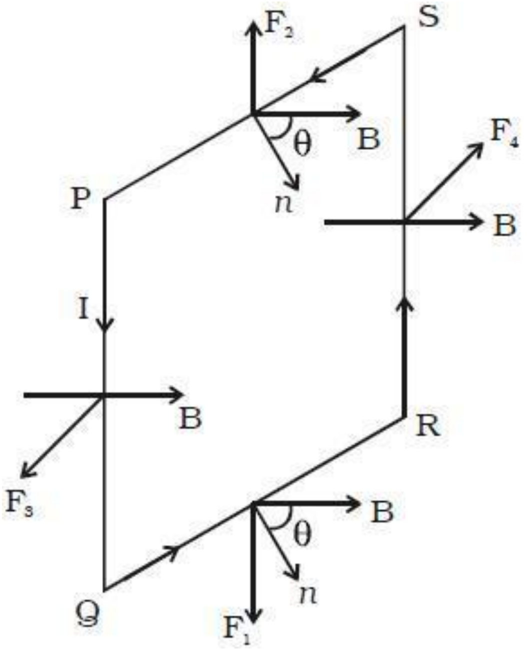
m = 60×10^{-6}Kg , B= 0.8 $\times10^{-4}$T, L = 30×10^{-2}, θ = 30^o, g = 10 m/s^2

$60\times10^{-6}\times10 = 0.8 \times10^{-4} \times (I) \times 30\times10^{-2}\times (1/2)$

I = 50 A

Current Loop in uniform Magnetic field

Let us consider a rectangular loop PQRS of length *l* and breadth *b* (Fig 3.24). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field. Force on the arm QR,



Force on the arm QR,

$$\vec{F_1} = \overrightarrow{I(QR)}\times\overline{B}$$

Since the angle between **I(QR)** and **B** is (90^o – θ),

Magnitude of the force F_1 = BIb sin (90^o – θ)

ie. F_1 = BIb cos θ

Force on the arm SP,

$$\vec{F_2} = \overrightarrow{I(SP)}\times\overline{B}$$

Since the angle between **I(SP)** and **B** is (90^o + θ),

Magnitude of the force F_2 = BIb cos θ

The forces F_1 and F_2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.

Force on the arm PQ,

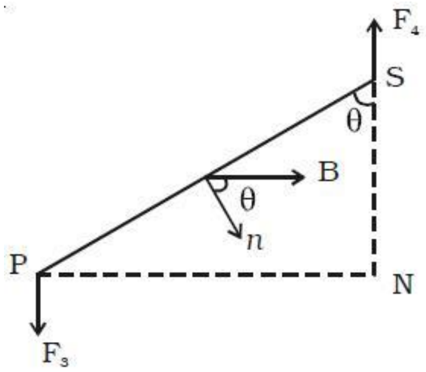
$$\vec{F} = \overrightarrow{I(PQ)}\times\overline{B}$$

Since the angle between I(PQ) and B is 90^o,
Magnitude of the force F_3 = BIL sin 90^o = BIL
F_3 acts perpendicular to the plane of the paper and outwards.

Force on the arm RS,

$$\overrightarrow{F_4} = \overrightarrow{I(RS)} \times \overline{B}$$

Since the angle between I(RS) and B is 90^o,
Magnitude of the force F_4 = BIL sin 90o = BIL



F_4 acts perpendicular to the plane of the paper and inwards.

The forces F_3 and F_4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple. Hence,
Torque = BIL × PN = BIL × PS × sin θ = BIL × b sin θ = BIA sin θ
If the coil contains N turns, τ = NBIA sin θ
So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.
The torques can be expressed as vector product of the magnetic moment of the coil and the magnetic field. We define the magnetic moment of the current loop as, **m** =N I **A** where the direction of the area vector **A** is given by the right-hand thumb rule and is directed into the plane of the paper in Figure. Then as the angle between **m** and **B** is θ
τ = **m** × **B**
The dimensions of the magnetic moment are [A] [L^2] and its unit is Am^2.
From equation we see

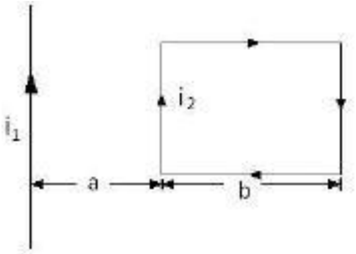
(i) the torque τ vanishes when **m** is either parallel or antiparallel to the magnetic field **B**.
(ii) This indicates a state of equilibrium as there is no torque on the coil (this also applies to any object with a magnetic moment **m**).
(iii) When **m** and **B** are parallel the equilibrium is a stable one. Any small rotation of the coil produces a torque which brings it back to its original position.
(iv) When they are antiparallel, the equilibrium is unstable as any rotation produces a torque which increases with the amount of rotation.
(v) The presence of this torque is also the reason why a small magnet or any magnetic dipole aligns itself with the external magnetic field.

Solved Numerical

Q) The arrangement is as shown below

(a) Find the potential energy of the loop

(b) Find the work done to increase the spacing between the wire and the loop a to 2a

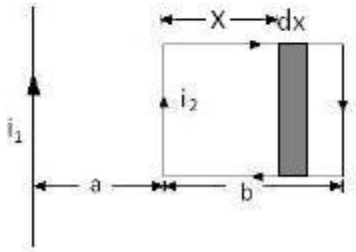


Solution:

(a) Magnetic field produced due to wire carrying current I_1 is inversely proportional to distance thus magnetic field associated with loop is not uniform. Consider small area of width dx and height L

Magnetic moment of a small element of the loop dM = i_2Ldx

The direction of the magnetic moment is perpendicular to the plane of the paper



Potential energy dU = -**dM . B**

Where B is the magnetic field at the position of this element

$$B = \frac{\mu_0}{4\pi}\frac{2I_1}{a+x}$$

$$dU = -\frac{\mu_0}{4\pi}2I_1I_2L\left(\frac{dx}{a+x}\right)$$

$$U = -\frac{\mu_0}{4\pi}2I_1I_2L\int_0^L\left(\frac{dx}{a+x}\right)$$

$$U = \frac{\mu_0}{4\pi}2I_1I_2L\log\left(\frac{a+b}{a}\right)$$

(b) Work done to increase the spacing between the wire and the loop from a to 2a

W = ΔU

$$U_i = -\frac{\mu_0}{4\pi} 2 I_1 I_2 L \log\left(\frac{a+b}{a}\right)$$

$$U_f = \frac{\mu_0}{4\pi} 2 I_1 I_2 L \log\left(\frac{2a+b}{2a}\right)$$

$$\Delta U = U_f - U_i = \frac{\mu_0}{4\pi} 2 I_1 I_2 L \log\left(\frac{2a+2b}{2a+b}\right)$$

Q) A rectangular coil of area 20 cm × 10 cm with 100 turns of wire is suspended in a radial magnetic field of induction 5 × 10-3 T. If the galvanometer shows an angular deflection of 150 for a current of 1mA, find the torsional constant of the suspension wire.

Solution:

n = 100, A = 20 cm × 10 cm = $2 \times 10^{-1} \times 10^{-1}$ m^2 , B = 5×10^{-3} T, , I = 1mA = 10^{-3}A,

θ = 15^0 = 15 (π/180) = π/12 , C = ?

nBIA = Cθ

$$C = \frac{nBIA}{\theta}$$

$$C = \frac{10\times5\times10^{-3}\times10^{-3}\times2\times10^{-1}\times10^{-1}}{\left(\frac{\pi}{12}\right)}$$

$$C = 3.82\times10^{-5}\, Nm\, rad^{-1}$$

Moving coil galvanometer

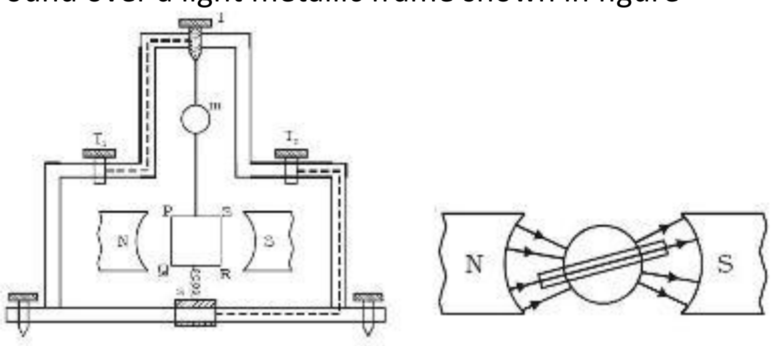
Moving coil galvanometer is a device used for measuring the current in a circuit.

Principle

Moving coil galvanometer works on the principle that a current carrying coil placed in a magnetic field experiences a torque.

Construction

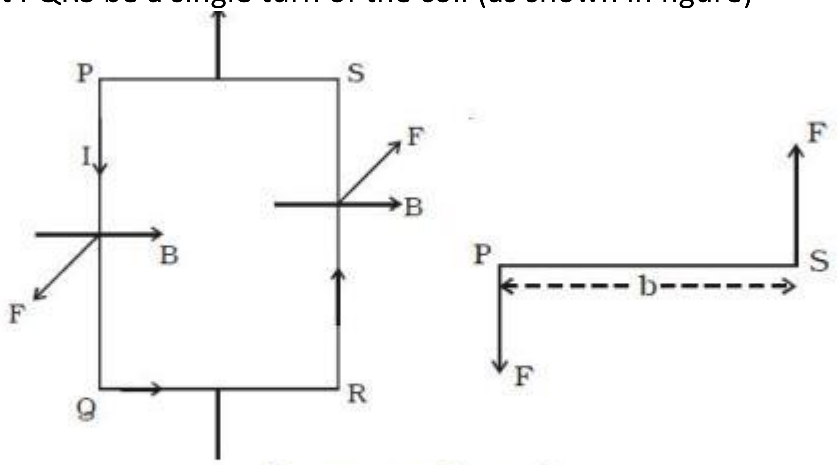
It consists of a rectangular coil of a large number of turns of thin insulated copper wire wound over a light metallic frame shown in figure



The coil is suspended between the pole pieces of a horse-shoe magnet by a fine phosphor – bronze strip from a movable torsion head. The lower end of the coil is connected to a hair spring (HS) of phosphor bronze having only a few turns. The other end of the spring is connected to a binding screw. A soft iron cylinder is placed symmetrically inside the coil. The hemispherical magnetic poles produce a radial magnetic field in which the plane of the coil is parallel to the magnetic field in all its positions as shown in figure. A small plane mirror (m) attached to the suspension wire is used along with a lamp and scale arrangement to measure the deflection of the coil.

Theory

Let PQRS be a single turn of the coil (as shown in figure)



Torque on the coil

A current I flows through the coil. In a radial magnetic field, the plane of the coil is always parallel to the magnetic field. Hence the sides QR and SP are always parallel to the field. So, they do not experience any force. The sides PQ and RS are always perpendicular to the field. PQ = RS = L, length of the coil and PS = QR = b, breadth of the coil

Force on PQ, F = BI (PQ) = BIL. According to Fleming's left hand rule, this force is normal to the plane of the coil and acts outwards.

Force on RS, F = BI (RS) = BIL.

This force is normal to the plane of the coil and acts inwards.

These two equal, oppositely directed parallel forces having different lines of action constitute a couple and deflect the coil. If there are *n* turns in the coil,

Torque of the deflecting couple = *N* BIL × b = *N*BIA

When the coil deflects, the suspension wire is twisted. On account of elasticity, a restoring couple is set up in the wire. This couple is proportional to the twist. If θ is the angular twist, then, moment of the restoring couple = Cθ

where C is the restoring couple per unit twist

At equilibrium, deflecting couple = restoring couple

$$NBIA = C\theta$$

$$I = \frac{C}{NBA}\theta$$

$$I = K\theta$$

Here K is the galvanometer constant.

$I \propto \theta$.

Since the deflection is directly proportional to the current flowing through the coil, the scale is linear and is calibrated to give directly the value of the current

We define the current sensitivity of the galvanometer as the deflection per unit current. current sensitivity is,

$$\frac{\theta}{I} = \frac{NAB}{C}$$

C is restoring couple per unit twist

Note current sensitivity is proportional to number of turns (N)

Conversion of galvanometer into an ammeter

A galvanometer is a device used to detect the flow of current in an electrical circuit. To measure current in circuit Galvanometer is connected in series. Because of following reasons Galvanometer cannot be used as Ammeter

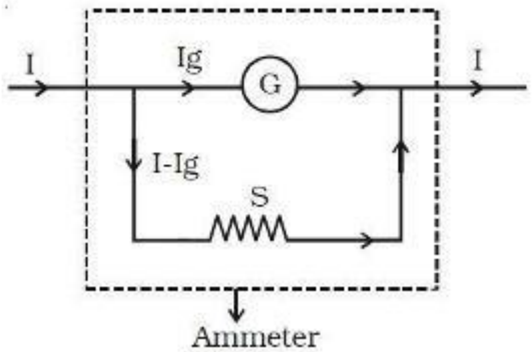
(i) Being a very sensitive instrument, a large current cannot be passed through the galvanometer, as it may damage the coil.

(ii) Galvanometer have resistance in few kilo-ohm resistance which get added to resistance of circuit as a result current in circuit changes.

However, a galvanometer is converted into an ammeter by connecting a low resistance in parallel with it. As a result, when large current flows in a circuit, only a small fraction of the current passes through the galvanometer and the remaining larger portion of the current passes through the low resistance.

The low resistance connected in parallel with the galvanometer is called shunt resistance. The scale is marked in ampere. The value of shunt resistance depends on the fraction of the total current required to be passed through the galvanometer.

Let I_g be the maximum current that can be passed through the galvanometer.



The current I_g will give full scale deflection in the galvanometer.

Galvanometer resistance = G. Shunt resistance = S

Current in the circuit = I

Current through the shunt resistance $I_s = (I - I_g)$

Since the galvanometer and shunt resistance are parallel, potential is common.

$I_g \cdot G = (I - I_g)S$

$$S = G\frac{I_g}{I - I_g}$$

The shunt resistance is very small because Ig is only a fraction of I. The effective resistance of the ammeter R_a is (G in parallel with S)

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

$$R_a = \frac{GS}{G+S}$$

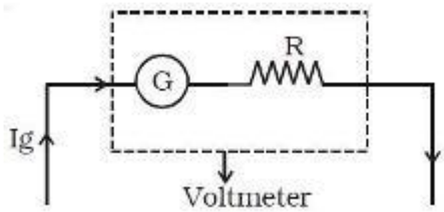
R_a is very low and this explains why an ammeter should be connected in series. When connected in series, the ammeter does not appreciably change the resistance and current in the circuit. Hence an ideal ammeter is one which has zero resistance.

Conversion of galvanometer into a voltmeter

Voltmeter is an instrument used to measure potential difference between the two ends of a current carrying conductor. A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The scale is calibrated in volt. The value of the resistance connected in series decides the range of the voltmeter.
Galvanometer resistance = G
The current required to produce full scale deflection in the galvanometer = Ig
Range of voltmeter = V Resistance to be connected in series = R
Since R is connected in series with the galvanometer, the current through the galvanometer,

$$I_g = \frac{V}{R+G}$$

$$R = \frac{V}{I_g} - G$$



From the equation the resistance to be connected in series with the galvanometer is calculated.
The effective resistance of the voltmeter is $R_v = G + R$
R_v is very large, and hence a voltmeter is connected in parallel in a circuit as it draws the least current from the circuit.
In other words, the resistance of the voltmeter should be very large compared to the resistance across which the voltmeter is connected to measure the potential difference. Otherwise, the voltmeter will draw a large current from the circuit and hence the current through the remaining part of the circuit decreases. In such a case the potential difference measured by the voltmeter is very much less than the actual potential difference. The error is eliminated only when the voltmeter has a high resistance. An ideal voltmeter is one which has infinite resistance.

Current loop as a magnetic dipole

Ampere found that the distribution of magnetic lines of force around a finite current carrying solenoid is similar to that produced by a bar magnet. This is evident from the fact that a compass needle when similar deflections moved around these two bodies show. After noting the close resemblance between these two, Ampere demonstrated that a simple current loop behaves like a bar magnet and put forward that all the magnetic phenomena is due to circulating electric current. This is Ampere's hypothesis.
The magnetic induction at a point along the axis of a circular coil carrying current is

$$B = \frac{\mu_0 NIa^2}{2\left(a^2 + x^2\right)^{3/2}}$$

The direction of this magnetic field is along the axis and is given by right hand rule. For points which are far away from the centre of the coil, *x>>a, a*^{2} is small and it is neglected. Hence for such points,

$$B = \frac{\mu_0 NIa^2}{2x^3}$$

If we consider a circular loop, n = 1, its area A = πa^2

$$B = \frac{\mu_0 IA}{2\pi x^3} \quad \text{....eq(1)}$$

The magnetic induction at a point along the axial line of a short bar magnet is

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

$$B = \frac{\mu_0}{2\pi}\frac{M}{x^3} \quad \text{...eq(2)}$$

Comparing equations (1) and (2), we find that
M = IA ...(3)
Hence a current loop is equivalent to a magnetic dipole of moment M = IA
The magnetic moment of a current loop is defined as the product of the current and the loop area. Its direction is perpendicular to the plane of the loop.

The magnetic dipole moment of a revolving electron

According to Neil Bohr's atom model, the negatively charged electron is revolving around a positively charged nucleus in a circular orbit of radius *r*. The revolving electron in a closed path constitutes an electric current. The motion of the electron in anticlockwise direction produces conventional current in clockwise direction. Current, i = e/T
where T is the period of revolution of the electron. If f is the orbital velocity of the electron, then

$$T = \frac{2\pi r}{v}$$

$$i = \frac{ev}{2\pi r}$$

Due to the orbital motion of the electron, there will be orbital magnetic moment μ_l

$\mu_l = i$ A, where A is the area of the orbit.

$$\mu_l = \frac{ev}{2\pi r}\pi r^2$$

$$\mu_l = \frac{evr}{2}$$

If m is the mass of the electron

$$\mu_l = \frac{e}{2m}(mvr)$$

mvr is the angular momentum (l) of the electron about the central nucleus.

$$\mu_l = \frac{e}{2m}(l)$$

$$\frac{\mu_l}{l} = \frac{e}{2m}$$

is called gyromagnetic ratio and is a constant. Its value is 8.8×10^{10} C kg^{-1}. Bohr hypothesized that the angular momentum has only discrete set of values given by the equation.

l = nh/2π ...(2) where n is a natural number

and h is the Planck's constant = 6.626×10^{-34} Js.

From above two equations for μ_l we get

$$\mu_l = \frac{e}{2m}\frac{nh}{2\pi} = \frac{neh}{4\pi m}$$

The minimum value of magnetic moment is

$$(\mu_l)_{\min} = \frac{eh}{4\pi m}, n = 1$$

Value of (eh/4πm) is called Bohr magneton

By substituting the values of e, h and m, the value of Bohr magneton is found to be 9.27×10^{-24} Am^2

In addition to the magnetic moment due to its orbital motion, the electron possesses magnetic moment due to its spin. Hence the resultant magnetic moment of an electron is the vector sum of its orbital magnetic moment and its spin magnetic moment.

Solved Numerical

Q) A moving coil galvanometer of resistance 20 Ω produces full scale deflection for a current of 50 mA. How you will convert the galvanometer into (i) an ammeter of range 20 A and (ii) a voltmeter of range 120 V.

Solution:

G = 20 Ω ; Ig = 50 x 10^{-3} A ; I = 20 A, S = ?

V = 120 V, R = ?

(i) Ammeter

$$S = G\frac{I_g}{I - I_g}$$

$$S = 20\frac{50\times10^{-3}}{20 - 50\times10^{-3}}$$

$$S = 0.05\,\Omega$$

A shunt of 0.05 Ω should be connected in parallel

(ii) Voltmeter

$$R = \frac{V}{I_g} - G$$

$$R = \frac{120}{50\times10^{-3}} - 20$$

$$R = 2380\,\Omega$$

A resistance of 2380 Ω should be connected in series with the galvanometer

Q) The deflection in a galvanometer falls from 50 divisions to 10 divisions when 12 Ω resistance is connected across the galvanometer. Calculate the galvanometer resistance.

Solution:

θ_1 = 50 divs, θg = 10 divs, S = 12Ω G = ?

I α θ_1

Ig α θg

In a parallel circuit potential is common.

$$G\cdot I_g = S\left(I - I_g\right)$$

$$G = \frac{S\left(I - I_g\right)}{I_g}$$

$$G = \frac{12\left(50 - 10\right)}{10}$$

$$G = 48\,\Omega$$

Q) In a hydrogen atom electron moves in an orbit of radius 0.5 Å making 1016 revolutions per second. Determine the magnetic moment associated with orbital motion of the electron.

Solution:

r = 0.5 Å = 0.5×10^{-10} m, n = 1016 s^{-1}

Orbital magnetic moment μ_l = i.A ...(1)

i = e/T

I = e.f ...(2)

A = πr^2 ...(3)

substituting equation (2), (3) in (1)

$\mu_l = e.n.\ \pi r^2$

$= 1.6 \times 10\text{-}19 \times 1016 \times 3.14\ (0.5 \times 10^{-10})^2$

$= 1.256 \times 10^{-23}$

$\therefore \mu_l = 1.256 \times 10^{-23}$ Am^2

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